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Davydov-Chaban Hamiltonian within the formalism of deformation-dependent effective mass for the Kratzer potential

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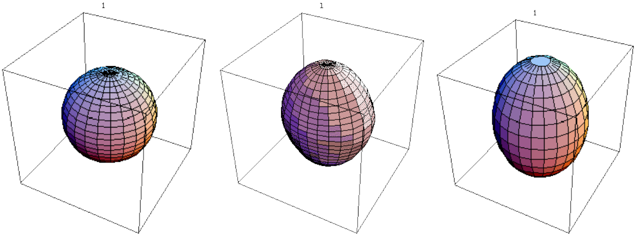
Introduction : Geometrical Collective Model

Geometrical collective (Bohr-Mottelson) Hamiltonian has 5 degrees of freedom, namely the two shape variables β and γ and the three Euler angles :

A. Bohr, and B. R. Mottelson, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. 27 (1953) No. 16.

$$H\Psi(\beta,\gamma,\theta_1,\theta_2,\theta_3) = E\Psi(\beta,\gamma,\theta_1,\theta_2,\theta_3)$$

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{1}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right] + V(\beta,\gamma)$$



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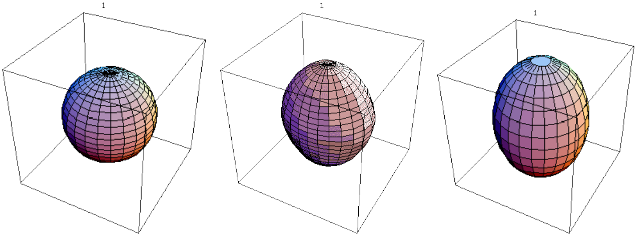

$$H_B = \beta \text{vibration} \quad + \quad \gamma \text{vibration} \quad + \quad \text{rotation} \quad + \quad \text{potential}$$

- Imposing a certain value for the γ shape variable, one reaches the γ-rigid version of the collective model which is interesting by itself due to its description of the basic rotation-vibration coupling.

Davydaov-Chaban model

The γ-rigid Hamiltonian for $\gamma = \pi/6$:

$$H\Psi(\beta, \theta_1, \theta_2, \theta_3) = E\Psi(\beta, \theta_1, \theta_2, \theta_3)$$

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^3}\frac{\partial}{\partial\beta}\beta^3\frac{\partial}{\partial\beta} - \frac{1}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right] + V(\beta)$$

- **Z(4) :** A. S. Davydov, and A. A. Chaban, Nucl. Phys. 20 (1960) 499.
  - ⇝ Generally appropriate for non-axial even-even nuclei, which are soft with respect to β vibrations of nuclear surface.

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Purpose :

- Construct a Davydov-Chaban Hamiltonian by allowing the nuclear mass to depend on the deformation, in accordance with the formalism of position-dependent effective mass.
  [B. Bagchi, A. Banerjee, C. Quesne and V. M. Tkachuk, J. Phys. A 38, 2929, 2007]

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Davydov-Chaban (DC) Hamiltonian

with Deformation-Dependent Mass (DDM)

- Allowing the nuclear mass to depend on the deformation : $B_0 \to B = \frac{B_0}{f(\beta)^2}$

  ⇝ **Position dependent effective mass : O. von Roos, Phys. Rev. B 27, 7547 (1983).**
- DC Hamiltonian with DDM ⇔ to a modified DC hamiltonian with different metric and different effective potentials.

$$\left[-\frac{1}{2}\frac{\sqrt{f}}{\beta^3}\frac{\partial}{\partial\beta}\beta^3 f \frac{\partial}{\partial\beta}\sqrt{f} + \frac{f^2}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} + v_{eff}\right]\psi = \epsilon\psi$$

with, $v_{eff} = v(\beta,\gamma) + \frac{1}{4}(1-\delta-\lambda)f\,\nabla^2 f + \frac{1}{2}\left(\frac{1}{2}-\delta\right)\left(\frac{1}{2}-\lambda\right)(\nabla f)^2$

- Considering the total wave function of the form $\psi(\beta,\Omega) = \chi(\beta)\phi(\Omega)$, the associated Schrödinger equation is separated in two parts :

$$\left[-\frac{1}{2}\frac{\sqrt{f}}{\beta^3}\frac{\partial}{\partial\beta}\beta^3 f \frac{\partial}{\partial\beta}\sqrt{f} + \frac{f^2}{2\beta^2}\Lambda + v_{eff}\right]\chi(\beta) = \epsilon\chi(\beta),$$

$$\left[\frac{1}{4}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} - \Lambda\right]\phi(\Omega) = 0.$$

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Z(4)-DDM Solution :

Solution of angular part

- In the case of $\gamma = \pi/6$, the angular momentum term can be written as

$$\sum_{k=1,2,3} \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} = 4(Q_1^2 + Q_2^2 + Q_3^2) - 3Q_1^2.$$

- angular equation has been solved by Meyer-ter-Vehn **Ref. [Nuclear Physics A 249 (1975) 111-140]**, with the results

$$\Lambda = L(L+1) - \frac{3}{4}\alpha^2,$$

$$\phi(\Omega) = \phi^L_{\mu,\alpha}(\Omega) = \sqrt{\frac{2L+1}{16\pi^2(1+\delta_{\alpha,0})}} \left[\mathcal{D}^{(L)}_{\mu,\alpha}(\Omega) + (-1)^L \mathcal{D}^{(L)}_{\mu,-\alpha}(\Omega)\right],$$

- we introduce the wobbling quantum number $n_w = L - \alpha$, the eigenvalues of the angular part are written as

$$\Lambda = L(L+1) - \frac{3}{4}(L - n_w)^2,$$

- Kratzer potential
  - $v(\beta) = -\frac{1}{\beta} + \frac{\beta_0}{2\beta^2}$,
- Deformation function
  - $f(\beta) = 1 + a\beta, \qquad a << 1.$

⇝ Solving the radial equation through Asymptotic Iteration Method (AIM)

- the generalized formula of the energy eigenvalues

$$\epsilon_{n_\beta n_w L} = \frac{1}{2}\left[k_0 + \frac{a^2}{4} - \left(\frac{k_{-1} + a n_\beta^2 + a\eta(1+2n_\beta)}{2(\eta+n_\beta)}\right)^2\right].$$

- The radial wave function :

$$R_{n_\beta L}(\beta) = C_{n_\beta L}\, a^{-\eta}\, 2^{n_\beta+\kappa n}\,(1+t)^\eta\,(1-t)^{-\kappa n-\eta-n_\beta}\, P_{n_\beta}^{(-1-2(n_\beta+\kappa n+\eta),2\eta-1)}(t),$$

- The parametrization used :

$k_0 = a^2\left[4 + 2\Lambda + 2(\frac{1}{2} - \delta)(\frac{1}{2} - \lambda)3(1 - \delta - \lambda)\right]$	$\eta = \frac{1}{2}(1 + \sqrt{1 + 4k_{-2}})$
$k_{-1} = a(3 + 2\Lambda + \frac{3}{2}(1 - \lambda - \delta) - 2$	$\kappa = -\frac{1}{2}\left(1 + n_\beta + \eta + \frac{k_{-2} - \frac{k_{-1}}{a}}{n_\beta+\eta}\right)$
$k_{-2} = \frac{3}{4} + \Lambda + \beta_0$	$t = \frac{-1+a\beta}{1+a\beta}$

$$C_{n_\beta L} = \left[\frac{a^{1+2\eta}}{2}\,\frac{(1+2n_\beta+2\eta+2\kappa n)(1+2\kappa n)}{(\eta+n_\beta)}\,\frac{\Gamma(1+n_\beta+2\kappa n)\Gamma(n_\beta+1)}{\Gamma(n_\beta+2\eta+2\kappa n)\Gamma(n_\beta+2\eta)}\right]^{\frac{1}{2}}.$$

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Numerical results :

Energy spectrum

- The energy ratios $R_{n_\beta,n_w,L}$ are defined by : $R_{n_\beta,n_w,L} = \frac{\epsilon_{n_\beta,n_w,L} - \epsilon_{0,0,0}}{\epsilon_{0,0,2} - \epsilon_{0,0,0}}$,
- The bands in the present model are classified by the following quantum numbers :
  - For gsb : $n_\beta = 0$ and $n_w = 0$;
  - For β band : $n_\beta = 1$ and $n_w = 0$;
  - For γ band : $n_\beta = 0$ and $n_w = 2$ for even L levels and $n_\beta = 0$ and $n_w = 1$ for odd L levels.
- We determine the optimal values of the free model's parameters by making use of the quality measure :

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(E_i(Exp) - E_i(th))^2}{(N-1)E(2_g^+)}}.$$

- The values of free parameters fitted to the experimental data

nucleus	β_0	a
^{192}Pt	49.8	0.002
^{194}Pt	71.6	0.004
^{196}Pt	51.0	0.006

Numerical results :

The energy spectra of the $^{192,194,196}Pt$ isotopes

	^{192}Pt				^{194}Pt				^{196}Pt			
	Exp	K	Ref 1	Ref 2	Exp	K	Ref 1	Ref 2	Exp	K	Ref 1	Ref 2
$R_{0,0,4}$	2.479	2.451	2.439	2.396	2.470	2.506	2.415	2.406	2.465	2.455	2.513	2.481
$R_{0,0,6}$	4.314	4.129	3.787	3.834	4.298	4.334	3.835	3.902	4.290	4.144	3.709	3.701
$R_{0,0,8}$	6.377	5.844	5.773	5.761	6.392	6.306	5.880	5.896	6.333	5.877	5.579	5.559
$R_{0,0,10}$	8.624	7.472	7.350	7.484	8.672	8.280	7.573	7.713	8.558	7.528	6.914	6.932
$R_{1,0,0}$	3.776	3.768	3.397	3.537	3.858	3.806	3.706	3.809	3.192	3.124	2.954	2.977
$R_{1,0,2}$	4.547	4.472	4.995	5.162	4.603	4.555	5.409	5.493	3.828	3.844	4.308	4.364
$R_{1,0,4}$		5.506	7.002	7.113		6.265	6.265	7.490		4.904	6.238	6.280
$R_{0,2,2}$	1.935	1.900	1.653	1.664	1.894	1.926	1.661	1.676	1.936	1.902	1.646	1.643
$R_{0,1,3}$	2.910	2.714	2.302	2.345	2.809	2.786	2.332	2.378	2.852	2.719	2.249	2.252
$R_{0,2,4}$	3.795	4.548	4.229	4.200	3.743	4.806	4.268	4.273	3.636	4.566	4.179	4.150
$R_{0,1,5}$	4.682	4.748	4.342	4.360	4.563	5.034	4.402	4.446	4.526	4.769	4.243	4.227
$R_{0,2,6}$	5.905	6.785	6.358	6.466		7.434	6.524	6.645	5.644	6.830	6.041	6.049
$R_{0,1,7}$	6.677	6.637	6.065	6.215		7.255	6.235	6.392		6.681	5.737	5.754
$R_{0,2,8}$	8.186	8.640	9.163	9.203		9.764	-	9.508	7.730	8.717	8.564	8.573
rms		0.500	0.614	0.593		0.390	0.543	0.515		0.576	0.682	0.683

- **Ref1 : P. Buganu, R. Budaca, PRC 91 (2015) 014306** ⇝ Z(4)-Sextic for k=5 .
- **Ref2 : R. Budaca, P. Buganu, M. Chabab, A. Lahbas, M. Oulne, Ann Phys 375 (2016) 65.** ⇝ Z(4)-Sextic for k=20

Numerical results :

B(E2) Transition rates

- The $B(E2)$ transition rates are given by

$$B(E2; L_i\alpha_i \to L_f\alpha_f) = \frac{5}{16\pi}\frac{|\left\langle L_f\alpha_f \;||\; T^{(E2)} \;||\; L_i\alpha_i \right\rangle|^2}{(2L_i+1)}$$

- The quadrupole operator for triaxial nuclei around $\gamma = \pi/6$ is given by

$$T_M^{(E2)} = t\beta\left[\mathcal{D}^{(2)}_{M,0}(\theta_i)\cos(\gamma - \frac{2\pi}{3}) + \frac{1}{\sqrt{2}}(\mathcal{D}^{(2)}_{M,2}(\theta_i) + \mathcal{D}^{(-2)}_{M,0}(\theta_i))\sin(\gamma - \frac{2\pi}{3})\right]$$

- The general expression for E2 transition probabilities is

$$B(E2; L_i\alpha_i \to L_f\alpha_f) = \frac{5}{16\pi}\frac{t^2}{2}\frac{1}{(1+\delta_{\alpha_i,0})(1+\delta_{\alpha_f,0})}[(L_i2L_f|\alpha_i2\alpha_f)$$

$$+(L_i2L_f|\alpha_i - 2\alpha_f) + (-1)^{L_f}(L_i2L_f|\alpha_i - 2 - \alpha_f)]^2 \times [I_\beta(n_i, L_i, \alpha_i, n_f, L_f, \alpha_f)]^2$$

$$I_\beta(n_i, L_i, \alpha_i, n_f, L_f, \alpha_f) = \int_0^\infty \beta\xi_{n_i,L_i,\alpha_i}(\beta)\xi_{n_f,L_f,\alpha_f}(\beta)\beta^3 d\beta$$

Numerical results :

B(E2) transition rates of the ^{192,194,196}Pt isotopes

nuleus	$\frac{4g\to 2g}{2g\to 0g}$	$\frac{6g\to 4g}{2g\to 0g}$	$\frac{8g\to 6g}{2g\to 0g}$	$\frac{10g\to 8g}{2g\to 0g}$	$\frac{2\gamma\to 2g}{2_1\to 0g}$	$\frac{2\gamma\to 0g}{2g\to 0g}$ $\times 10^3$	$\frac{0_\beta\to 2g}{2g\to 0g}$	$\frac{2_\beta\to 0g}{2g\to 0g}$ $\times 10^3$	rms
^{192}**Pt**	1.56(12)	1.23(55)			1.91(16)	9.5(9)			
	1.58	2.38	3.32	4.61	1.60	0.0	0.70	17.36	**0.2966**
^{194}**Pt**	1.73(13)	1.36(45)	1.02(30)	0.69	1.81(25)	5.9(9)	0.01		
	1.56	2.31	3.19	4.41	1.58	0.0	0.67	25.08	**0.6239**
^{196}**Pt**	1.48(3)	1.80(23)	1.92(23)			0.4	0.07(4)	0.06(6)	
	1.63	2.58	3.86	5.91	1.65	0.0	0.90	23.16	**0.3751**

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Conclusions

- A new solution for the Davydov-Chaban Hamiltonian within the DDM for the Kratzer potential is proposed, called Z(4)-DDM Kratzer.
- From the mathematical point of view : this work is achieved through the use of Iteration Asymptotic Method IAM, exact analytical expressions for the spectra and wave functions have been obtained ;
- From the physics point of view : the numerical realization of this model consisted of calculating energy spectra and transition probabilities of ^{192,194,196}Pt isotopes using Kratzer as collective potential compared to experimental data and some models calculations.
- Experimental intraband and interband transitions rates are slightly underestimated.
- The predicted energy spectra are in good agreement with the experimental data for the studied nuclei.

Thank you for your attention !