Binding and Bonding of Cascade-hyperons in Nuclei

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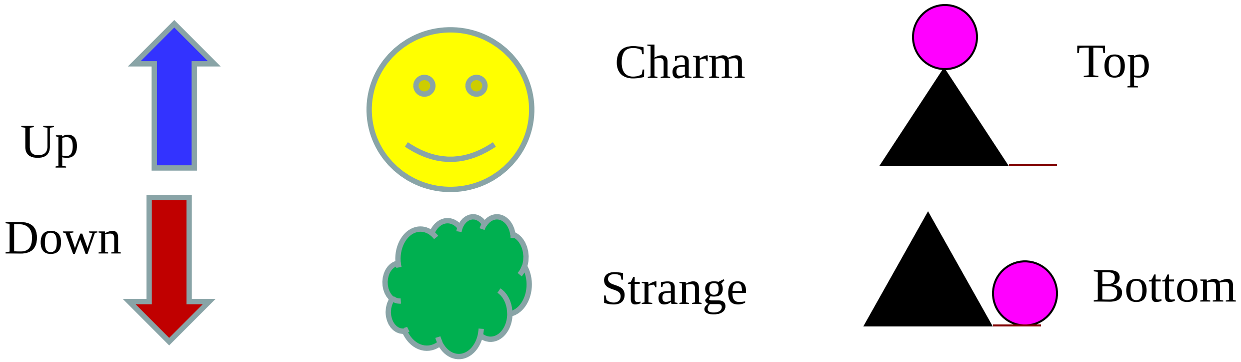
Outline

- Building blocks of a nucleus:
  - Quarks
  - Neutrons, Protons, Hyperons
  - Hypernucleus
- Binding energy of a nucleus
  - Microscopic calculation
  - Macroscopic calculation
- Binding & Neutron drip-point
  - Normal nucleus
  - Hypernucleus
- Hyperon-hyperon Bonding in Hypernuclei:
  - Bonding of two Lambda hyperons
  - Bonding of two Cascade hyperons
- Summary

Building Blocks of a Nucleus

A quark is an elementary particle and a fundamental constituent of matter.

Six flavors of quarks**.**



Up

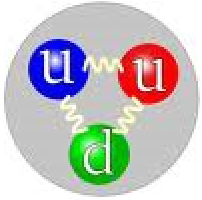
Down

Charm

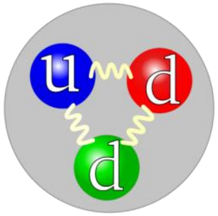
Strange

Top

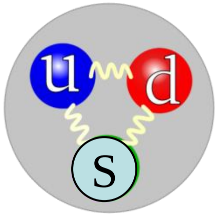
Bottom



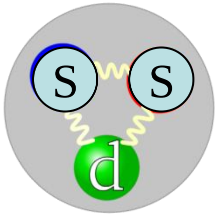
p=(**uud**)



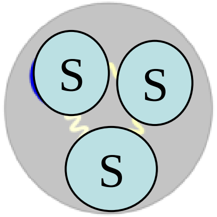
n=(**udd**)



Λ^0=(**uds**)



Ξ^-=(**dss**)



Ω^-=(**sss**)

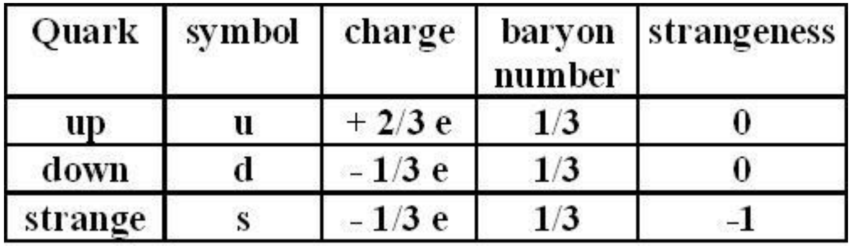
Nucleons

Hyperons

- Nucleons and hyperons both are called **baryons**.

Charge & Mass of Different Hyperons

A hyperon contains at least one strange **S** quark



Quark	symbol	charge	baryon number	strangeness
up	u	+ 2/3 e	1/3	0
down	d	- 1/3 e	1/3	0
strange	s	- 1/3 e	1/3	-1

Particle	Symbol	Quarks	Charge (e)	Strangeness (S)	Mass (MeV/c^2)	Mean Lifetime (s)	Commonly Decays to
Proton **(free)**	p	uud	+1	0	938.272	> 6.62×10^{36}	-
Neutron **(free)**	n	u d d	0	0	939.57	879.6±0.8	$p^+ + e^- + \bar{\nu}_e$
Lambda	Λ^o	u d s	0	-1	1 115.683(6)	2.60×10^{-10}	$p + \pi^-$ or, $n^o + \pi^o$
Cascade-zero	Ξ^o	u s s	0	-2	1 314.86(20)	(2.90±0.09) $\times 10^{-10}$	$\Lambda^o + \pi^o$
Cascade-minus	Ξ^-	d s s	-1	-2	1 321.71(7)	(1.639±0.015) $\times 10^{-10}$	$\Lambda^o + \pi^-$

Normal Nucleus and Hypernucleus

Normal Nucleus = Neutrons (N) + Protons (Z)

Symbol: ^{A}Z

Z= Total Charge = charge of Protons = Z_c
A = Number of Neutrons + Number of Protons

Hypernucleus = Neutrons (N)+ protons (Z) + Hyperons (Y)

Symbol: $^{A}_{Y}Z$

Z= Total Charge = Charge of Protons + Charge of hyperons
= $Z_c + Z_Y$

A = Number of Neutrons + Number of Protons + Number of Hyperons

How to Find the Binding Energy of a Nucleus?

❖Relativistic-mean-field (RMF) calculations:

- Can provide insight into the nucleus (structure, binding energy, excited states etc.)
- Sensitive to NN and NY effective interactions.
- Can calculate one nucleus at a time.
- Difficult to pursue near the driplines (convergence problems).

J. Schaffner J, C.B. Dover, A. Gal , C. Greiner, D.J. Millener and H. Stöcker, Ann. Phys. NY 235 (1994) 35.
S. Banik, M. Hempel, D. Bandyopadhyay, 2014 ApJS 214 (2014) 22.
A. Gal, E. V. Hungerford, D. J. Millener, Rev. Mod. Phys. 88 (2016) 035004.
K.A.Maslov, E.E.Kolomeitsev, D.N.Voskresensky, Nuclear Physics A950, (2016) 64.
Yu-Ting Rong, Shan-Gui Zhou, arXiv:2103.10706v1 [nucl-th].

❖A properly constructed mass formula:

- ➢ Can provide a quick check on the RMF calculations
- ➢ Can be used for a wider mass region from light to heavy - beyond the domain of RMF.
- ➢ Can calculate a large number of nuclei in a very short time.
- ➢ Can be used along with other programs that need nuclear masses in-situ computation.

C. Samanta et al., JPG32 (2006) 363.; C. Samanta, EPJ Web of Conferences 182, 02107 (2018).
A.S. Botvina, J. Pochodzalla, PRC 76 (2007) 024909.
N. Buyukcizmeci, A. S. Botvina, J. Pochodzalla, and M. Bleicher, Phys. Rev. C 88, (2013) 014611 .
N Buyukcizmeci, AS Botvina, R Ogul, AIP Conference Proceedings 1815 (2017), 060003.
N. Buyukcizmeci, A. S. Botvina, A. Ergun, R. Ogul, and M. Bleicher, Phys. Rev. C 98, (2018) 064603.

Variation of Λ, Ξ^- & ΛΛ Separation Energies with A

Binding energy: From a single mass formula for both normal and hypernuclei

$$B(A,Z) = 15.777A - 18.34A^{2/3} - 0.71Z(Z-1)/A^{1/3} - 23.21(N-Z_c)^2/[(1+e^{-A/17})A]$$
$$+n_Y[0.0335m_Y - 27.8 - 48.7|S|/A^{2/3}] + \delta$$

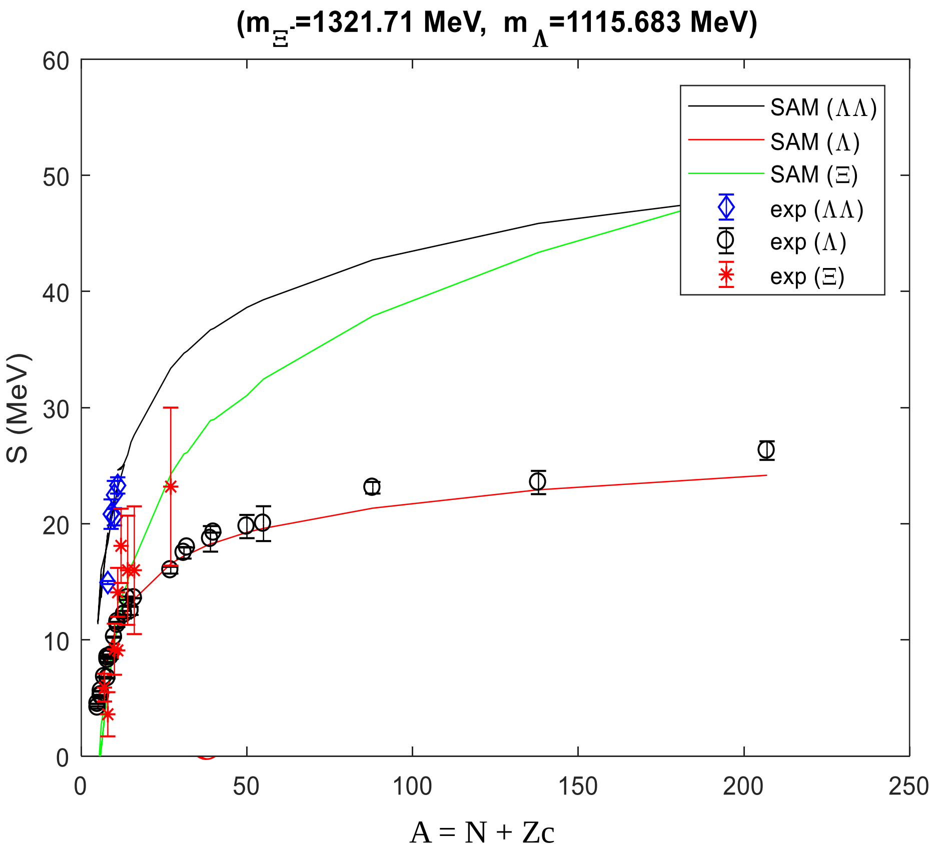
Normal Nucleus:
$A = N + Zc$
$Z = Zc$

Hypernucleus
= Core (Normal nucleus) + hyperon(s)
$A = N + Zc + n_Y$
$Z = Zc + n_Y * q_Y$

The hyperon separation energy S from a hypernucleus is defined as:

$$S = B(A, Z)_{hyper} - B(A-n_Y, Z_c)_{core}$$

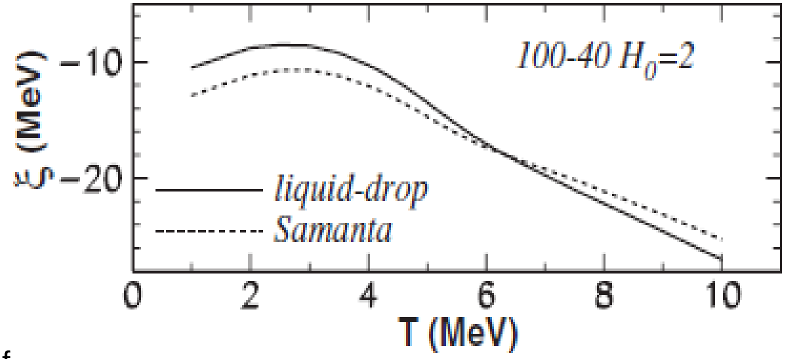
C. Samanta, EPJ Web of Conferences 182, (2018) 02107



Mass formulas used in the calculation of production of hypernuclei in multifragmentation

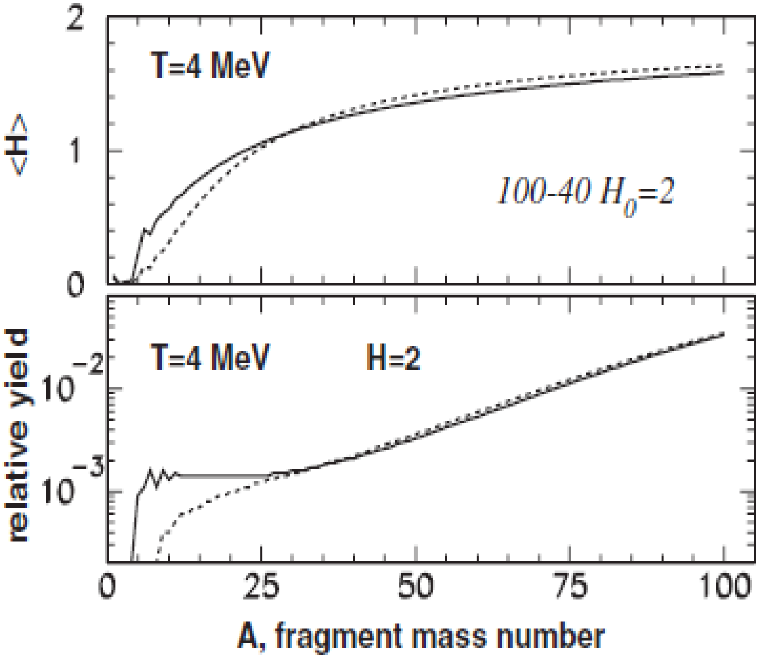
A.S. Botvina, J. Pochodzalla, PRC 76 (2007) 024909

S-Chemical potential



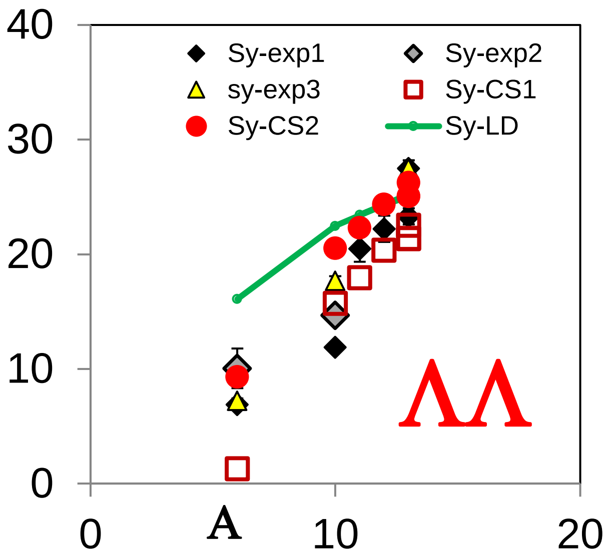
Av. No. of Λ-hypeons in fragments

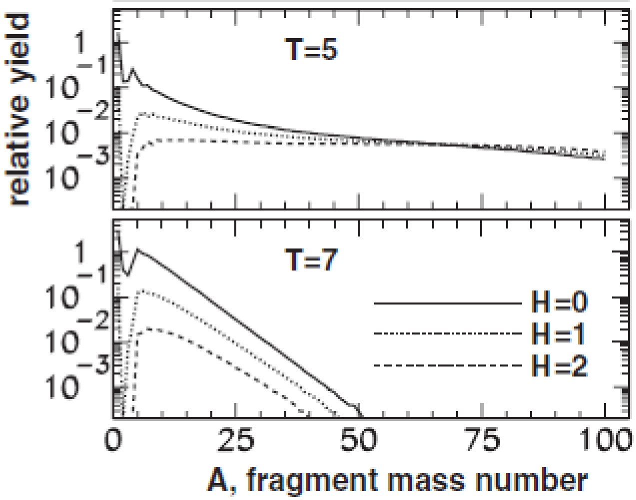
Yield of fragments with two Λ-hyperons at T=4 MeV



$S_{\Lambda\Lambda}$

(MeV)

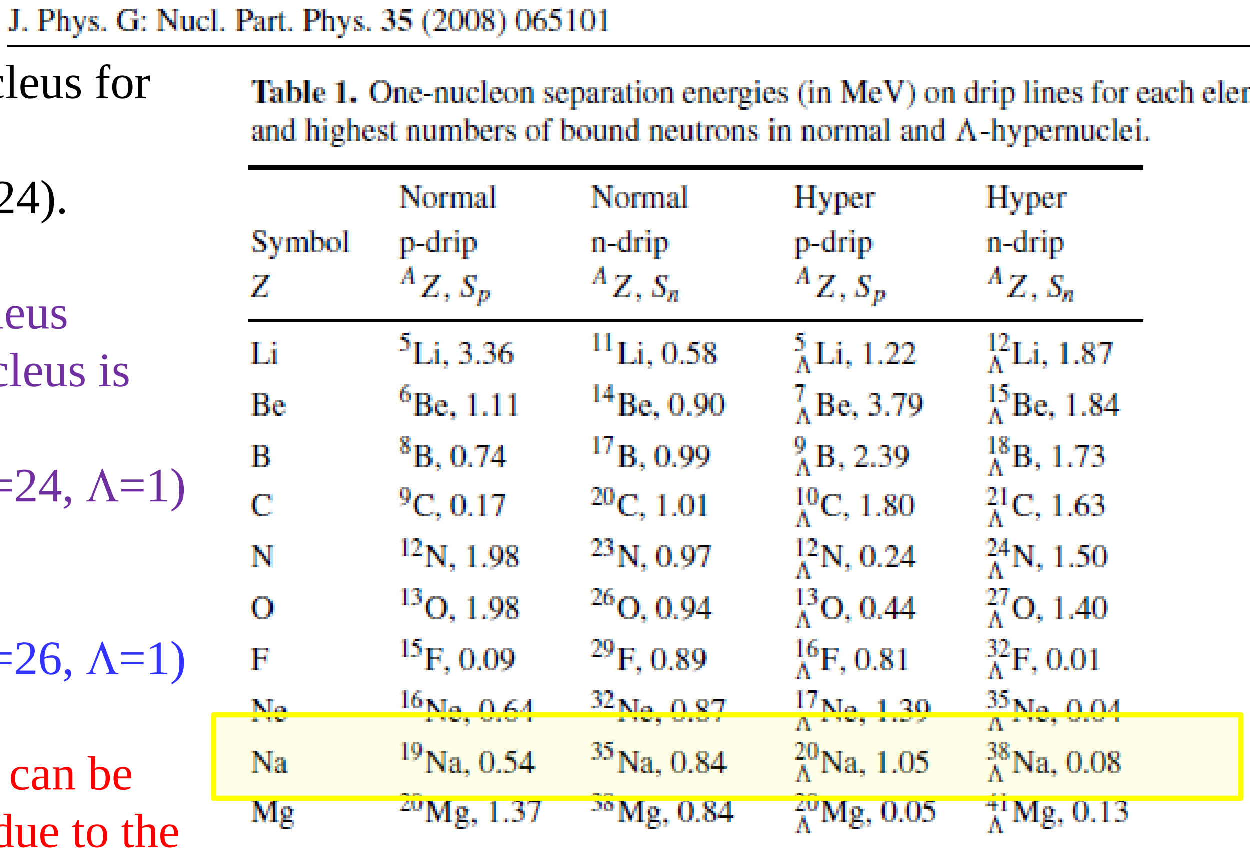




SMM= Statistical Multifragmentation model

Effects of Addition of Lambda

A Lambda-particle makes a nucleus more bound, and can change the neutron-, proton-drip points by creating bound nucleus beyond the normal drip lines.



J. Phys. G: Nucl. Part. Phys. **35** (2008) 065101 C Samanta *et al*

Table 1. One-nucleon separation energies (in MeV) on drip lines for each element with the lowest and highest numbers of bound neutrons in normal and Λ-hypernuclei.

Symbol Z	Normal p-drip $^{A}Z, S_p$	Normal n-drip $^{A}Z, S_n$	Hyper p-drip $^{A}Z, S_p$	Hyper n-drip $^{A}Z, S_n$
Li	^{5}Li, 3.36	^{11}Li, 0.58	$^{5}_{\Lambda}$Li, 1.22	$^{12}_{\Lambda}$Li, 1.87
Be	^{6}Be, 1.11	^{14}Be, 0.90	$^{7}_{\Lambda}$Be, 3.79	$^{15}_{\Lambda}$Be, 1.84
B	^{8}B, 0.74	^{17}B, 0.99	$^{9}_{\Lambda}$B, 2.39	$^{18}_{\Lambda}$B, 1.73
C	^{9}C, 0.17	^{20}C, 1.01	$^{10}_{\Lambda}$C, 1.80	$^{21}_{\Lambda}$C, 1.63
N	^{12}N, 1.98	^{23}N, 0.97	$^{12}_{\Lambda}$N, 0.24	$^{24}_{\Lambda}$N, 1.50
O	^{13}O, 1.98	^{26}O, 0.94	$^{13}_{\Lambda}$O, 0.44	$^{27}_{\Lambda}$O, 1.40
F	^{15}F, 0.09	^{29}F, 0.89	$^{16}_{\Lambda}$F, 0.81	$^{32}_{\Lambda}$F, 0.01
Ne	^{16}Ne, 0.64	^{32}Ne, 0.87	$^{17}_{\Lambda}$Ne, 1.39	$^{35}_{\Lambda}$Ne, 0.04
Na	^{19}Na, 0.54	^{35}Na, 0.84	$^{20}_{\Lambda}$Na, 1.05	$^{38}_{\Lambda}$Na, 0.08
Mg	^{20}Mg, 1.37	^{38}Mg, 0.84	$^{20}_{\Lambda}$Mg, 0.05	$^{41}_{\Lambda}$Mg, 0.13

Normal drip nucleus for Z=11 is:
^{35}Na (Z=11, N=24).

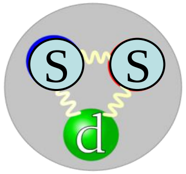
For Λ hypernucleus neutron-drip nucleus is not:
$^{36}{}_{\Lambda}$Na (Z=11, N=24, Λ=1)

Instead it is:
$^{38}{}_{\Lambda}$Na (Z=11, N=26, Λ=1)
i.e.,
2 extra neutrons can be accommodated due to the addition of a Λ.

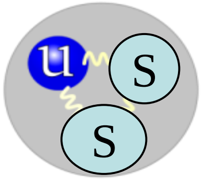
The exact drip-point may change due to the deformation and other parameters near the drip line.

Cascade Hypernuclei

Symbol	Makeup	Rest mass (MeV/c²)	Q (e)	S	Mean lifetime (s)	Commonly decays to
Λ^0	**uds**	**1115.68**	**0**	**−1**	**2.60×10^{-10}**	**$p^+ + \pi^-$ or $n^0 + \pi^0$**
Ξ^0	**uss**	**1 314.86**	**0**	**−2**	**$(2.90\pm0.09)\times10^{-10}$**	**$\Lambda^0 + \pi^0$**
Ξ^-	**dss**	**1 321.71**	**−1**	**−2**	**$(1.639\pm0.015)\times10^{-10}$**	**$\Lambda^0 + \pi^-$**



$\Xi^-=($**dss**$)$



$\Xi^o=($**uss**$)$

Symbol: $^{A}_{Y}Z$; Z= Total Charge = Charge of Protons + Charge of hyperons; Y= No. of hyperons

A = No. of protons + No. of Neutrons + No. of hyperons

For a hypernucleus with 3 protons and 1 Λ^o hyperon: **Total charge= Z = 3 + 0 =2**

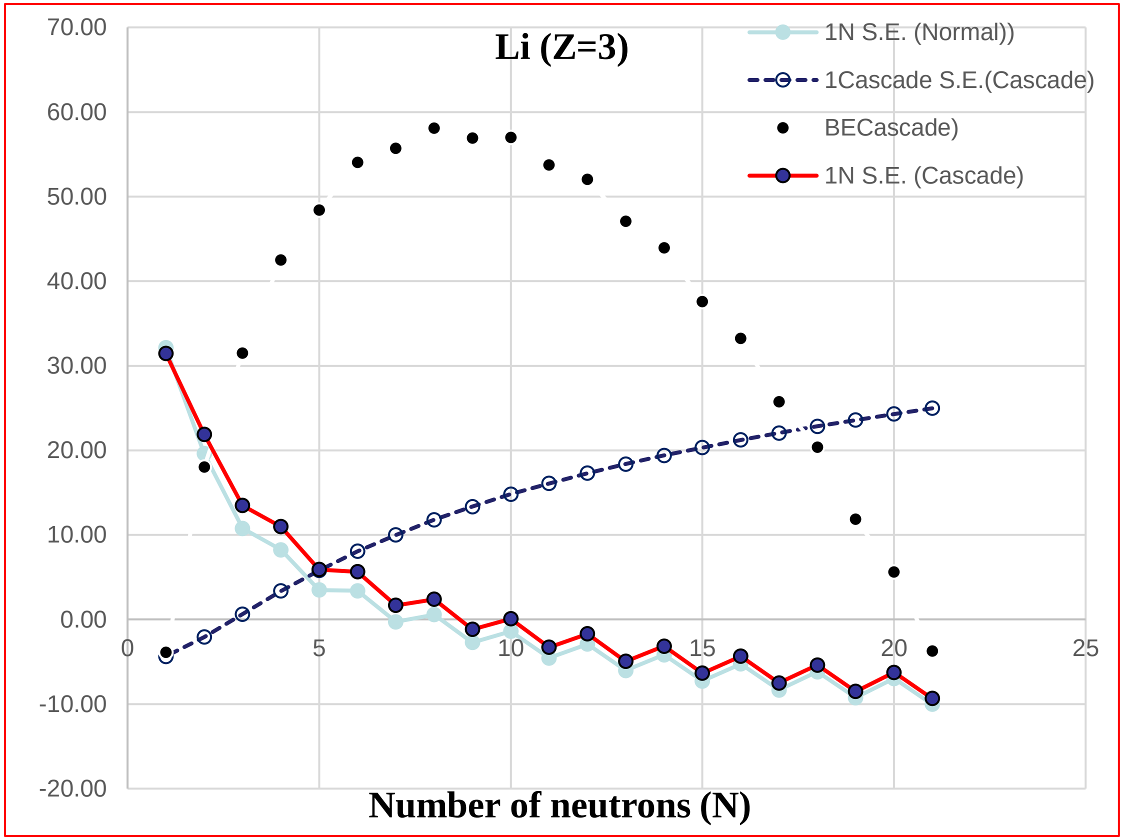
For a hypernucleus with 3 protons and 1 Ξ^- hyperon: **Total charge= Z = 3-1 =2**

Elements are named according to their total charge.

A normal nucleus with 3 protons and 4 neutrons is called, ^{7}Li

A hypernucleus with 3 protons, 4 neutrons and 1 Ξ^- hyperon => $^{8}_{\Xi^-}$He

Cascade Hypernuclei Beyond Normal Drippoint



- Normal nucleus with N= 7 **(^{10}Li)** is not bound as the one-neutron separation energy is less than zero.
- But, the Ξ^- hypernucleus with N=7 is bound, A = 7 + 3 + 1=11, Z = 3-1 =2 => **($^{11}{}_{\Xi}$–He).**
- **Both normal and Ξ^- hypernuclei do not exist for N=9 (as one neutron separation energy <0)**
- Drip point for Normal nucleus: N=8, Z = 3, A = 8 + 3 = 11 => **(^{11}Li).**
- Drip point for Ξ^- hypernucleus: N= 10, Z = 3-1 = 2, A = 10 +3 +1 => **($^{14}{}_{\Xi}$–He)**

Exotic Nucleus

Elements are named according to their total charge.

- Z =1, N =1 is bound => ^{2}H
- Z =1, N =1, Ξ^- = 1 is bound
- Total number of baryons: A = 1 + 1 + 1 = 3
- Charge of the resultant hypernuleus is: Z = 1 - 1= 0

=> ^{3}n

- Z=1, N=1, Ξ^- = 2 is bound
- Total number of baryons: A = 1 + 1 + 2 = 4

Charge of the resultant hypernuleus is Z= 1 - 2 = -1!

What should be its name?

How Two Λ-Hyperons Behave in a ΛΛ-Hypernucleus?

C Samanta, TA Schmitt, AIP Conference Proceedings 2130 (2019) 040004

Λ–Λ Bonding in a Double-Lambda Hypernucleus:

$$\Delta B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) - 2B_{\Lambda}({}^{A-1}_{\Lambda}Z)$$

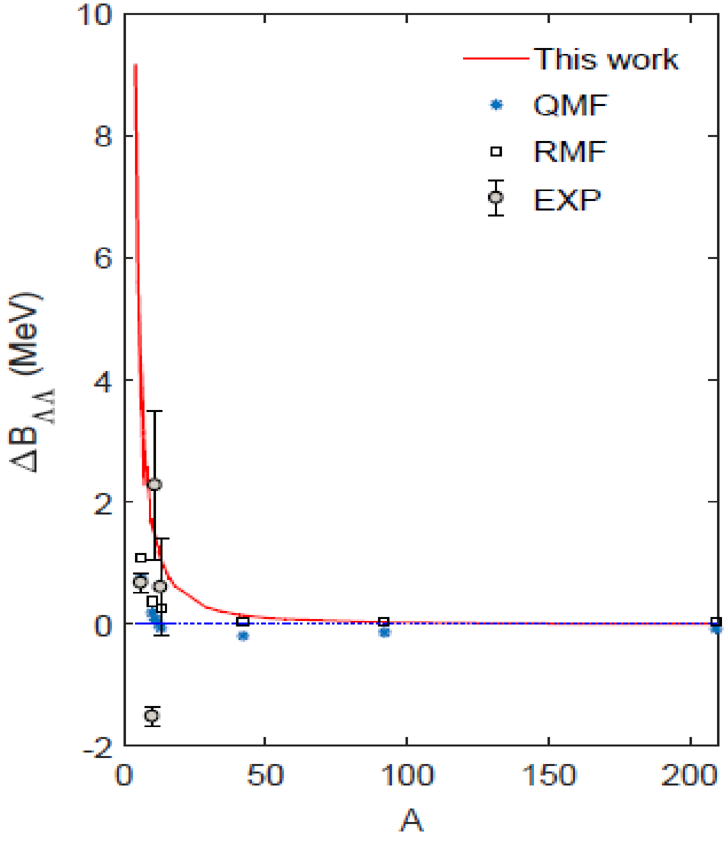


FIG. 4: Plot of ΛΛ-bond energy ($\Delta B_{\Lambda\Lambda}$) versus the mass number A.

The Λ–Λ bonding inside a double-Lambda hypernucleus diminishes with increasing neutron and proton numbers.

How Two Ξ^--Hyperons Behave in a $\Xi\Xi$-Hypernucleus?

Normal nucleus: A = Zc + N

Zc = no. of protons, N = and no. of neutrons

Adding one Ξ^- hyperon => Single Cascade hypernucleus: A = Zc + N + Ξ^-

Adding two Ξ^- hyperons => Double Cascade hypernucleus: A = Zc + N + Ξ^-

Ξ^- Ξ^- Bonding in a Double- Ξ^- Hypernucleus:

$$\Delta B_{\Xi\Xi}({}^{A}{}_{\Xi\Xi}Zc) = B_{\Xi\Xi}\,(A_{\Xi\Xi}Zc) - 2B_{\Xi}\,({}^{A-1}{}_{\Xi}Zc)$$

The $\Xi\Xi$ – bonding energy is found to diminish with the increasing number of neutrons and protons in hypernuclei.

Summary

- Hyperons are Baryons with Strange quark in addition to Up and Down quarks.
- A single generalized mass formula gives a rough and quick estimation of the binding, bonding and hyperon separation energies for normal and hypernuclei.
- Addition of Λ and Ξ^- hyperons makes a nucleus more bound and sometimes shifts the neutron drip points to higher neutron numbers.
- $\Lambda\Lambda$ and $\Xi^-\Xi^-$ bonding diminish with increasing nucleon numbers in a nucleus.
- There could be truly exotic bound nucleus with one proton, two neutrons and two cascade-minus (Ξ^-) hyperons and its total charge will be -1!

teşekkür ederim

Thank you!

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Extra slides

What is Strange about Hyperons?

Production: $\pi^- + p \rightarrow K^+ + \Lambda^0 + \pi^-$ (cross sections ~millibarns!)

$(\bar{u}\, d) + (uud) \rightarrow (u\, \bar{s}) + (uds) + (\bar{u}\, d)$ Strangeness is conserved

$\pi^- + p \neq K^- + \Lambda^0 + \pi^+$ not produced!

$(\bar{u}\, d) + (uud) = (\bar{u}\, s) + (uds) + (u\, \bar{d})$ Strangeness is not conserved

Although charge is conserved in both cases

Gell-Mann & Pais:

Strangeness: conserved in strong production processes

But Strangeness is violated in weak decays of hyperons!

Decay: $\Lambda^0(uds) \rightarrow \pi^-(\bar{u}\, d) + p\, (uud)$

$K^0\, (ds) \rightarrow \pi^+ (u\, \bar{d}) + \pi^- (\bar{u}\, d)$

Life-times characteristic of weak interaction(~10^{-10}Sec)