

# Gamow-Teller resonances and beta decay half-lives of nuclei at finite temperature

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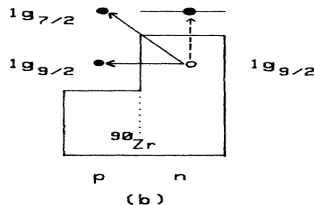
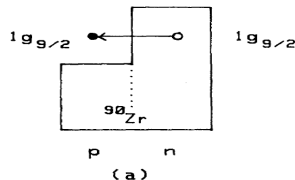
# Outline

- 1 Introduction and motivation
  - Spin-isospin excitations in nuclei
- 2 Microscopic models
  - Relativistic finite temperature proton-neutron QRPA
- 3 Numerical results
  - Gamow-Teller excitations at finite temperature
  - $\beta$ -decay half-lives at finite temperature
- 4 Conclusions and perspectives
- 5 Acknowledgments

Image by Andy Sproles, Oak Ridge National Laboratory.

# Introduction and motivation: spin-isospin excitations

- ✓ The spin-isospin resonances can be induced by isospin lowering ( $\tau_-$ ) or raising ( $\tau_+$ ) operators.
  - Isobaric analog states (IAS):  
 $\Delta L = \Delta J = \Delta S = 0$ ,
  - Gamow-Teller resonance (GTR):  
 $\Delta L = 0, \Delta J = \Delta S = 1$ ,
  - Spin monopole ( $J = 0^-$ ), dipole ( $J = 1^-$ ) and quadrupole ( $J = 2^-$ ) states.
- ✓ Their properties are important to understand the nuclear structure:
  - Spin and isospin properties of the effective nuclear interaction  
M. Bender et.al., PRC 65, 054322 (2002), H. Liang et.al., PRL 101 (2008) 122502.,
  - They can be used to predict neutron skin thickness of nuclei  
D. Vretenar et.al., Phys.Rev.Lett. 91, 262502 (2003).



(a) Fermi (b) Gamow-Teller transitions.  
F. Osterfeld, Rev. Mod. Phys., 64, 491–557 (1992).

# Introduction and motivation: spin-isospin excitations

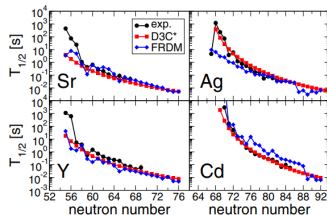
✓ ... and nuclear weak interaction processes in stellar environments:

- Calculation of the  $\beta$ -decay rates of r-process nuclei  
J.Engel et.al., Phys. Rev. C 60, 014302 (1999);  
T. Marketin et.al., Phys. Rev. C 93, 025805 (2015).
- Electron capture cross sections and rates  
K. Langanke et.al., Phys. Rev. Lett. 90, 241102 (2003);  
A. L. Cole et.al., Phys. Rev. C 86, 015809 (2012).
- and charged-current neutrino-nucleus reactions  
N. Paar et.al., Phys. Rev. C 77, 024608 (2008);  
N. Paar et. al., Phys. Rev. C 87, 025801 (2013).

... which take place at finite temperature.

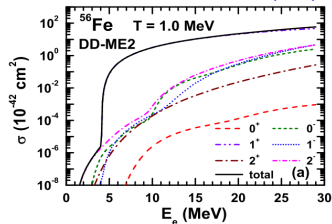
Therefore, accurate determination of the Gamow-Teller excitations as well as the nuclear properties is quite important for the astrophysical processes.

Self-consistent mean-field theories (HF, Q(RPA)) are standing as the prominent tools for calculations.



Beta decay half lifes.

T. Marketin, et.al., PRC 93, 025805 (2016).

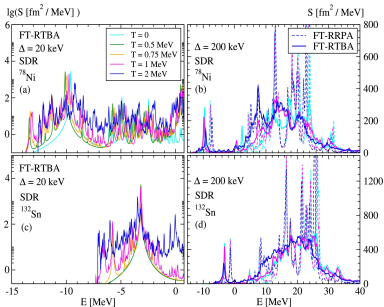
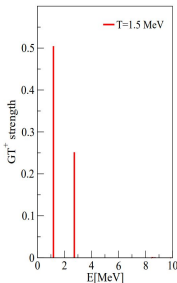
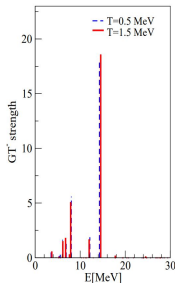


Electron capture cross sections are calculated using FT-PNRPA.

Y.F. Niu et.al., PRC 83, 045807 (2011).

# Overview of recent works

- ✓ Derivation of the FT-QRPA equations: H. M. Sommermann, Ann. Phys. (NY) 151, 163 (1983).
- ✓ The extended QRPA: O. Civitarese and M. Reboiro, Phys. Rev. C 63, 034323 (2001).
- ✓ Shell model+RPA: K. Langanke, E. Kolbe, and D. J. Dean, Phys. Rev. C 63, 032801(R) (2001).
- ✓ Thermo-Field-Dynamics (TFD) formalism: Alan A. Dzhirov et.al., Phys. Rev. C 81, 015804 (2010)
- ✓ Skyrme-TQRPA: Alan A. Dzhirov et.al., Phys. Rev. C 94, 015805 (2016).
- ✓ FT-RRPA: Y.F. Niu et.al., Phys. Rev. C 83, 045807 (2011).
- ✓ The relativistic (quasiparticle) time-blocking approximation: H. Wibowo and E. Litvinova, Phys. Rev. C 100, 024307 (2019).
- ✓ The proton-neutron relativistic RPA (pn-RRPA) beyond the one-loop approximation: E. Litvinova, C. Robin, H. Wibowo, Physics Letters B, 800, 135134 (2020).



Left: Temperature dependence of the Gamow-Teller strength for <sup>74</sup>Ge. Right: strength distribution in <sup>78</sup>Ni and <sup>132</sup>Sn at various temperature.

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- ✓ The extended QRPA: [O. Civitarese and M. Reboiro, Phys. Rev. C 63, 034323 \(2001\).](#)
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## Goal

- Complete investigation is necessary to understand the effect of the temperature on the spin-isospin response of the open-shell nuclei using relativistic nuclear energy density functionals!
- The purpose of the present work is to investigate the finite temperature effects on the Gamow-Teller excitations and the beta-decay half-lives of nuclei by developing the self consistent finite temperature proton-neutron QRPA.

# Finite temperature effects on the nuclear properties

The relativistic nuclear energy density functional with density dependent meson-nucleon couplings are used in the calculations, and pairing correlations are taken into account in the BCS scheme. In the relativistic approach, we use a monopole pairing interaction for the finite temperature Hartree BCS calculations.

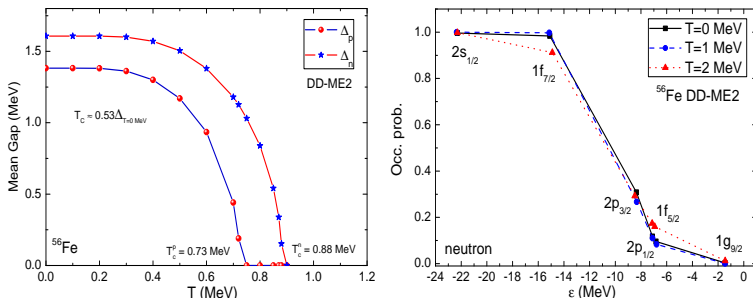


Figure 1: Left: Mean value of the neutron(proton) pairing gap as a function of the temperature for  $^{56}\text{Fe}$ . Right: occupation probabilities for neutron states as a function of temperature. The calculations are performed using DD-ME2 functional.

Temperature dependent Fermi-Dirac distribution function:  $f_i = [1 + \exp(E_i/k_B T)]^{-1}$

# Microscopic model: Finite temperature quasiparticle random phase approximation

The starting point in the Equation of Motion (EOM) method is the definition of a suitable excitation operator

$$\Gamma_{\nu}^{\dagger} = \sum_{a \geq b} X_{ab}^{\nu} a_a^{\dagger} a_b^{\dagger} - Y_{ab}^{\nu} a_b a_a + P_{ab}^{\nu} a_a^{\dagger} a_b - Q_{ab}^{\nu} a_b^{\dagger} a_a \quad (1)$$

two-quasiparticle creation/destruction operators and one-quasiparticle creation/ destruction operators. With  $|BCS\rangle$  as the approximate thermal vacuum the equation of motion can be written as:

$$\langle BCS | [\delta\Gamma, H, \Gamma_{\nu}^{\dagger}] | BCS \rangle = E_{\nu} \langle BCS | [\delta\Gamma, \Gamma_{\nu}^{\dagger}] | BCS \rangle \quad (2)$$

The FT-QRPA equations are derived as:

$$\tilde{A}_{abcd} = \sqrt{1 - f_a - f_b} A'_{abcd} \sqrt{1 - f_c - f_d} + (E_a + E_b) \delta_{ac} \delta_{bd}, \quad (3)$$

$$\tilde{B}_{abcd} = \sqrt{1 - f_a - f_b} B_{abcd} \sqrt{1 - f_c - f_d}, \quad (4)$$

$$\tilde{C}_{abcd} = \sqrt{f_b - f_a} C'_{abcd} \sqrt{f_d - f_c} + (E_a - E_b) \delta_{ac} \delta_{bd}, \quad (5)$$

$$\tilde{D}_{abcd} = \sqrt{f_b - f_a} D_{abcd} \sqrt{f_d - f_c}, \quad (6)$$

$$\tilde{a}_{abcd} = \sqrt{f_b - f_a} a_{abcd} \sqrt{1 - f_c - f_d}, \quad (7)$$

$$\tilde{b}_{abcd} = \sqrt{f_b - f_a} b_{abcd} \sqrt{1 - f_c - f_d}, \quad (8)$$

H. M. Sommermann, Ann. Phys. (NY) 151, 163 (1983).

E. Yüksel, G. Colò, E. Khan, Y.F. Niu, K. Bozkurt Phys. Rev. C. 96, 024303 (2017).



# Microscopic model: Finite temperature quasiparticle random phase approximation

The finite temperature QRPA equations can be combined into a single matrix as

$$\begin{pmatrix} \tilde{C} & \tilde{a} & \tilde{b} & \tilde{D} \\ \tilde{a}^+ & \tilde{A} & \tilde{B} & \tilde{b}^T \\ -\tilde{b}^+ & -\tilde{B}^* & -\tilde{A}^* & -\tilde{a}^T \\ -\tilde{D}^* & -\tilde{b}^* & -\tilde{a}^* & -\tilde{C}^* \end{pmatrix} \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix} = \hbar\omega \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix} \quad (9)$$

- $\tilde{A}$  and  $\tilde{B}$  (and their complex conjugates) describe the effects of two-quasiparticle excitations  $a^\dagger a^\dagger$  and  $a a$ .

$$\sum_{a \geq b} \left\{ |\tilde{X}_{ab}^\nu|^2 - |\tilde{Y}_{ab}^\nu|^2 + |\tilde{P}_{ab}^\nu|^2 - |\tilde{Q}_{ab}^\nu|^2 \right\} = 1, \quad (10)$$

The reduced transition probability is given as

$$\begin{aligned} B(EJ, \tilde{0} \rightarrow \nu) &= |\langle \nu || \hat{F}_J || \tilde{0} \rangle|^2 \\ &= \left| \sum_{c \geq d} \left\{ (\tilde{X}_{cd}^\nu + \tilde{Y}_{cd}^\nu)(v_c u_d + u_c v_d) \sqrt{1 - f_c - f_d} + (\tilde{P}_{cd}^\nu + \tilde{Q}_{cd}^\nu)(u_c u_d - v_c v_d) \sqrt{f_d - f_c} \right\} \langle c || \hat{F}_J || d \rangle \right|^2. \end{aligned} \quad (11)$$

# Finite temperature effects in the Gamow-Teller excitations of doubly-magic nuclei

- ✓ The calculations are performed for the  $^{78}\text{Ni}$  using the relativistic nuclear energy density functionals:

- The main GT strength is obtained around 9.8 MeV at  $T=0$  MeV.
- At  $T=0.5$  MeV, the main GT peaks are not affected, whereas the formation of the new excited states can be seen.

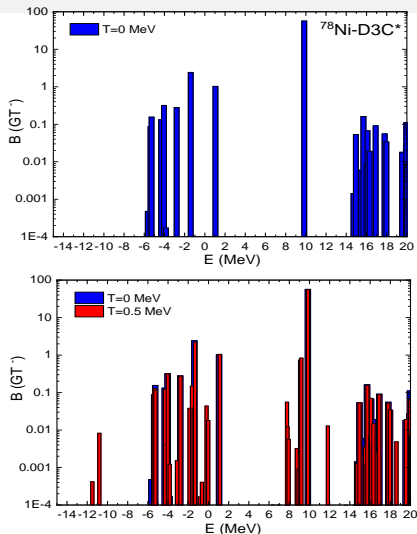


Figure 2: The GT excitations with increasing temperature.

# Finite temperature effects in the Gamow-Teller excitations of doubly-magic nuclei

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- The main GT strength is obtained around 9.8 MeV at  $T=0$  MeV.
- At  $T=0.5$  MeV, the main GT peaks are not affected, whereas the formation of the new excited states can be seen.
- At higher temperatures, the excited states starts to shift downwards. Besides, the strength becomes fragmented and the number of new excited states increases considerably.

- The occupation probabilities of the states below the Fermi level start to decrease, while the states above the Fermi level become populated,
- The new excitation channels become possible due to the smearing of the Fermi surface.
- The changes occur due to the temperature induced effects on the nuclear properties of nuclei and the residual particle-hole interaction.  $\uparrow V_{ph}$   $\downarrow$

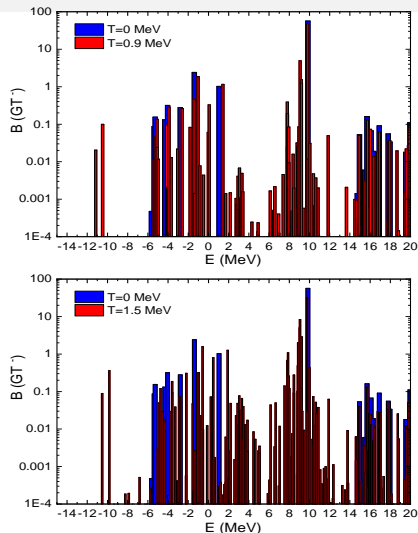


Figure 3: The GT excitations with increasing temperature.

# The role of the isoscalar pairing in the Gamow-Teller excitations

For the isoscalar pairing, we employ formulation with a short range repulsive Gaussian combined with a weaker longer range attractive Gaussian

$$V_{12} = -G_0^{is} \sum_{j=1}^2 g_j e^{-r_{12}^2/\mu_j^2} \prod_{S=1, T=0}, \quad (12)$$

- The isoscalar proton-neutron pairing contributes at the level of the residual interaction of the proton-neutron quasiparticle random phase approximation. Nuclei  $N \approx Z$  is sensitive to it. The isoscalar pairing strength?
- By increasing the isoscalar pairing strength, excitation energies and transition strengths decrease in the GTR region, while the excited states start to shift downward and strength increases in the low-energy region.
- The isoscalar pairing has an attractive nature: the excited state energies become smaller by increasing the isoscalar pairing strength.

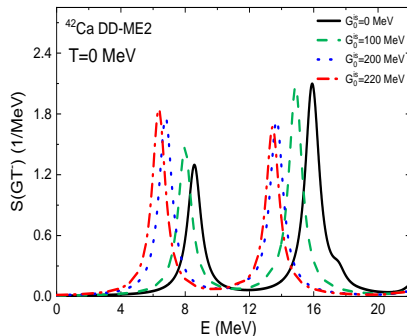


Figure 4: The GT excitations with increasing isoscalar pairing strength.

Reference: E. Yüksel, et. al., Phys. Rev. C 101, 044305 (2020).

# Finite temperature effects in the Gamow-Teller excitations

- Without the isoscalar pairing, the changes in the GT states are caused by the decrease of the isovector pairing effects and the softening of the repulsive  $ph$  interaction due to the temperature factors with increasing temperature.

- 1)  $T \uparrow \Rightarrow IV \text{ pairing} \downarrow \Rightarrow E_{conf} \downarrow$
- 2)  $T \uparrow \Rightarrow V_{res}^{ph(pp)} \downarrow \Rightarrow \text{Excited states} \downarrow$

- With the inclusion of the isoscalar pairing, the decrease in the excitation energies depends on the competition between the temperature and isoscalar pairing effects.
- The temperature reduces the impact of the both isoscalar and the isovector pairing.
- In a complete calculation the excited states slowly shift downwards in energy compared to the results without the isoscalar pairing with increasing temperature.

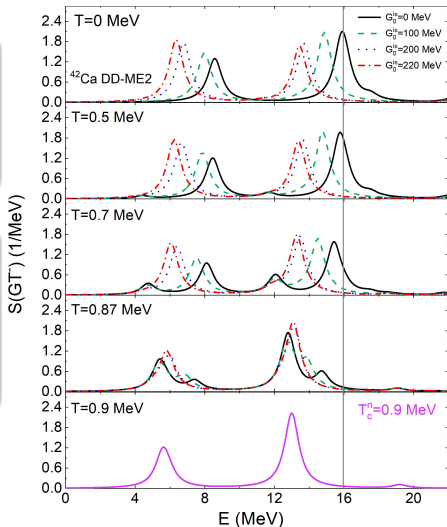


Figure 5: The GT excitations with increasing isospin and temperature.

# Numerical results - finite temperature effects

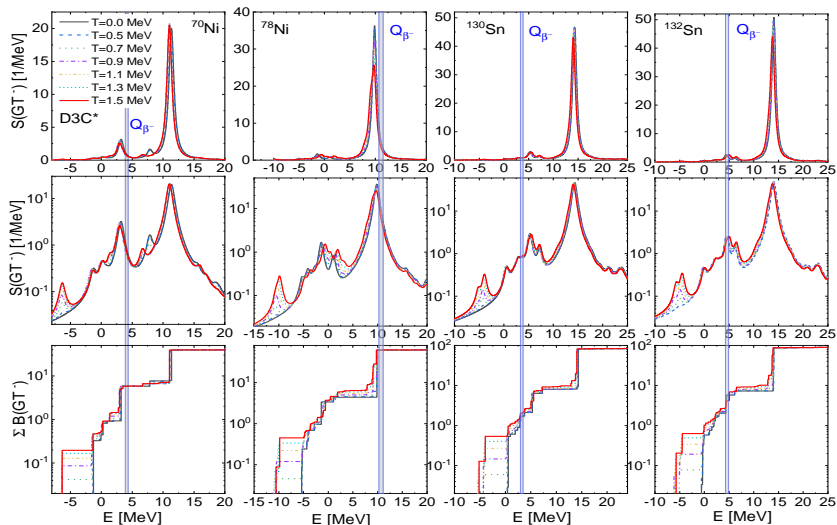


Figure 6: The Gamow-Teller strength in selected nuclei by increasing temperature.

# Calculation of the $\beta$ -decay half-lives

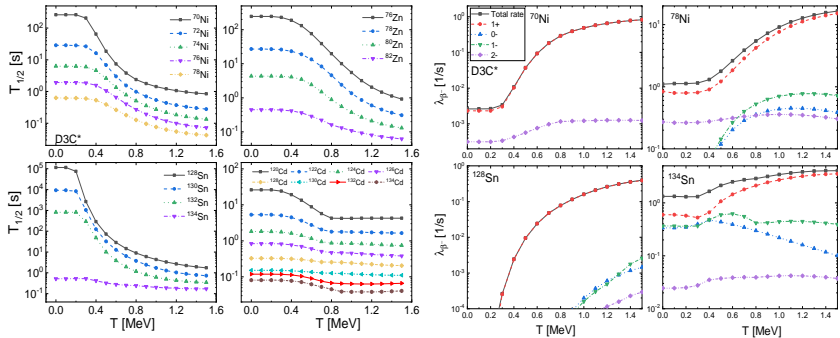


Figure 7: The  $\beta$ -decay half-lives (left) and contribution of the FF transitions to the  $\beta$ -decay rates (right) with increasing temperature.

- For the calculation of the  $\beta$ -decay half-lives, the  $GT^-$  excitations in the low-energy region and below the  $Q_{\beta-}$  values play the major role.
- By increasing temperature, the excited states shifts downwards and new excited states are obtained due to the unblocking mechanism of temperature. As a result of this,  $\beta$ -decay phase space increases, and half-lives decrease.
- We also find that the decrease in  $\beta$ -decay half-lives is more pronounced in nuclei with long half-lives, e.g.,  $^{70}\text{Ni}$  and  $^{128}\text{Sn}$ , whereas the decrease in the half-lives are less for nuclei with short half-lives.
- Contribution of the FF transitions increase with increasing temperature.

# Conclusions

- The relativistic finite temperature proton-neutron QRPA is developed and used in the calculations of the Gamow-Teller response and  $\beta$ -decay half-lives of nuclei.
- By increasing temperature, the Gamow-Teller excitation energies also start to shift downward by increasing temperature.
- Formation of the new low-energy strengths are obtained with the opening of the new excitation channels.
- The isoscalar pairing ( $T=0$ ) plays an important role in the low-energy region: increasing the strength and decreasing the excitation energy.
- Inclusion of the pairing in microscopic models is crucial for the proper description of the Gamow-Teller states below the critical temperatures.
- $\beta$ -decay half-lives decrease rapidly for nuclei with longer half-lives, whereas the changes are slight for nuclei with shorter half-lives.
- While the Gamow-Teller excitations dominate the behavior of the  $\beta$ -decay half-lives, the contribution of the first-forbidden (FF) transitions also increase with increasing temperature.

## Perspectives

- Proper treatment of nuclei at finite temperatures; particle-number projected FT-HFB or exact canonical treatment of pairing problem.
- Inclusion of beyond-mean field effects?
- Large-scale calculations at Finite Temperature? (By Ante Ravlic- Zagreb University)



# Acknowledgments



## Collaborators:

- **Ante Ravlic** (University of Zagreb, Croatia)
- **Nils Paar** (University of Zagreb, Croatia)
- **Yifei Niu** (Lanzhou University, China)
- **Gianluca Colò** (Università degli Studi di Milano and INFN)
- **Elias Khan** (IJCLab, Orsay, France)