Nucleon Densities of Lanthanum Isotopes Calculated by Skyrme and Gogny Models

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Introduction

- The Hartree-Fock (HF) codes are a useful tool to describe the ground states properties of the spherical nucleus. Spherical HF codes have been used for 12 years and are developed by deriving gradient iterations. In HF approach, equations are solved using an iteration, namely, harmonic oscillator or Wood-Saxon wave functions are recommended for wave function. The density $\rho(0)$ is generated based on these wave functions. Then, iterations are made between the intensity and $U(\rho)$ energy potential for φ wave function.

- The Skyrme force is the effective force for HF calculations. In this way, quantities, which are very important and can be measured experimentally, such as core radii, density distributions and surface thickness can be calculated. HF codes are applied to the specified Skyrme force by fitting the least squares method. It is also fitted to ground states correlations and nucleus excitation studies.

Skyrme

Skyrme forces are the best phenomenological force used to describe the ground states of nucleus. Skyrme interactions, including the interaction of three objects with two-body interaction is defined as;

$$\vec{V}_{\text{Skyrme}} = \sum_{i<j} \vec{V}(i,j) + \sum_{i<j<k} \vec{V}(i,j,k) \tag{1}$$

The first term defines two objects and the second term defines interaction of three bodies. Short-range two-body interaction is given as;

$$\begin{aligned}\vec{V}(i,j) = {} & t_0(1+x_0P_x)\delta(\vec{r}_i-\vec{r}_j) + \frac{1}{2}t_1(1+x_1P_x)\{\vec{p}_{12}^{\,'2}\delta(\vec{r}_i-\vec{r}_j)+\delta(\vec{r}_i-\vec{r}_j)\vec{p}_{12}^{\,2}\} \\ & + t_2(1+x_2P_x)\vec{p}_{12}^{\,'}\delta(\vec{r}_i-\vec{r}_j)\vec{p}_{12} + it_4\vec{p}_{12}^{\,'}\cdot\delta(\vec{r}_i-\vec{r}_j)(\vec{\sigma}_i+\vec{\sigma}_j)\times\vec{p}_{12}\end{aligned} \tag{2}$$

In this equation, δ is delta function, $\overrightarrow{p_{12}} = \pm\frac{1}{2} i\,(\overrightarrow{\nabla_1} - \overrightarrow{\nabla_2})$ is relative momentum operator acting on the wave functions moving to the right / left, P_x is spin exchange operator and σ is Pauli spin matrices. The interaction of two objects depends on seven parameters (t_0, t_1, t_2, x_0, x_1, x_2 and t_4). The three-body interaction is given as:

$$\vec{V}(i, j, k) = \frac{1}{6} t_3 (1 + x_3 P_x) \rho\left(\frac{r_i + r_j}{2}\right) \delta(\vec{r}_i - \vec{r}_j) \qquad (3)$$

ρ, the diagonal part of a total body density operator in the coordinate space and t_3 represent a parameter that characterizes the density dependence of the interaction.

Skyrme-HFB

Many properties of nuclei can be described in terms of a model comprised of independent particles moving in an average potential. The space dependence of this potential should closely follow the matter distribution from the Skyrme-HF method. Using this model, for independent particles, a single particle potential can be derived from the two-body interaction. This is done using a variational principle, with Slater determinants (of, for example, harmonic oscillator or Woods-Saxon wave functions) as trial wave functions.

Another type of HF method is the Hartree-Fock-Bogolyubov (HFB) method. Independent particles moving within an average potential are depicted as single particles Correlations between these particles and incomplete shells give the HFB theory. HFB-S and HFB-G methods were developed by using Skyrme and Gogny forces.

Gogny Forces

Another phenomenological force is the Gogny force which is a finite-range two-body interaction. The use of the Gogny interaction with the HF method yields a non-relativistic approach like the SHF method.

The Gogny force has been widely used in various mean-field calculations of nuclear structure. The parameters of he Gogny force were determined by mean-field calculations fitting to the experimental data of infinit nuclei and the properties of infinite nuclear matter. The effect of the three-body force is taken into account in both the Skyrme and Gogny forces through a density-dependent term, which is essential to describe various properties of nuclei and nuclear matter.

Nucleon Densities

The neutron and proton densities are given by:

$$\rho_q(\vec{r}) = \sum_{\beta \in q} w_\beta \psi_\beta(\vec{r})^+ \psi_\beta(\vec{r}) \tag{4}$$

where q indicates neutron or proton density and w_b denotes the occupation probability of the state β.

For the SHF method, Y_b represents the single-particle wave function of the state β.

Alternatively, for the SHFB method, Y_b represents the quasiparticle wave function of the quasiparticle state β. Using either method, the rms radii of neutron and proton densities can be evaluated using Eq. (4) and the following formula:

$$r_q = \left\langle r_q^2 \right\rangle^{1/2} = \left[\frac{\int r^2 \rho_q(r) dr}{\int \rho_q(r) dr} \right]^{1/2} \tag{5}$$

For the Skyrme-HF method, the nuclear charge density is obtained from the charge form factor by the inverse Fourier-Bessel transform:

$$\rho_c = \frac{1}{2\pi^2}\int k^2 j_0(kr)F_c(k)dk \tag{6}$$

where $F_c(k)$ is the charge form factor, and j_0 is the spherical Bessel function of the zeroth order. The rms radii of charge densities, also for the Skyrme-HF method, are defined by the following formula:

$$r_c = \left\langle r_c^2 \right\rangle^{1/2} = \left[\frac{\int r^2 \rho_c(r)dr}{\int \rho_c(r)dr}\right]^{1/2} \tag{7}$$

The charge radius for the Skyrme-HFB method is obtained from the proton radius by approximating for the proton charge distribution:

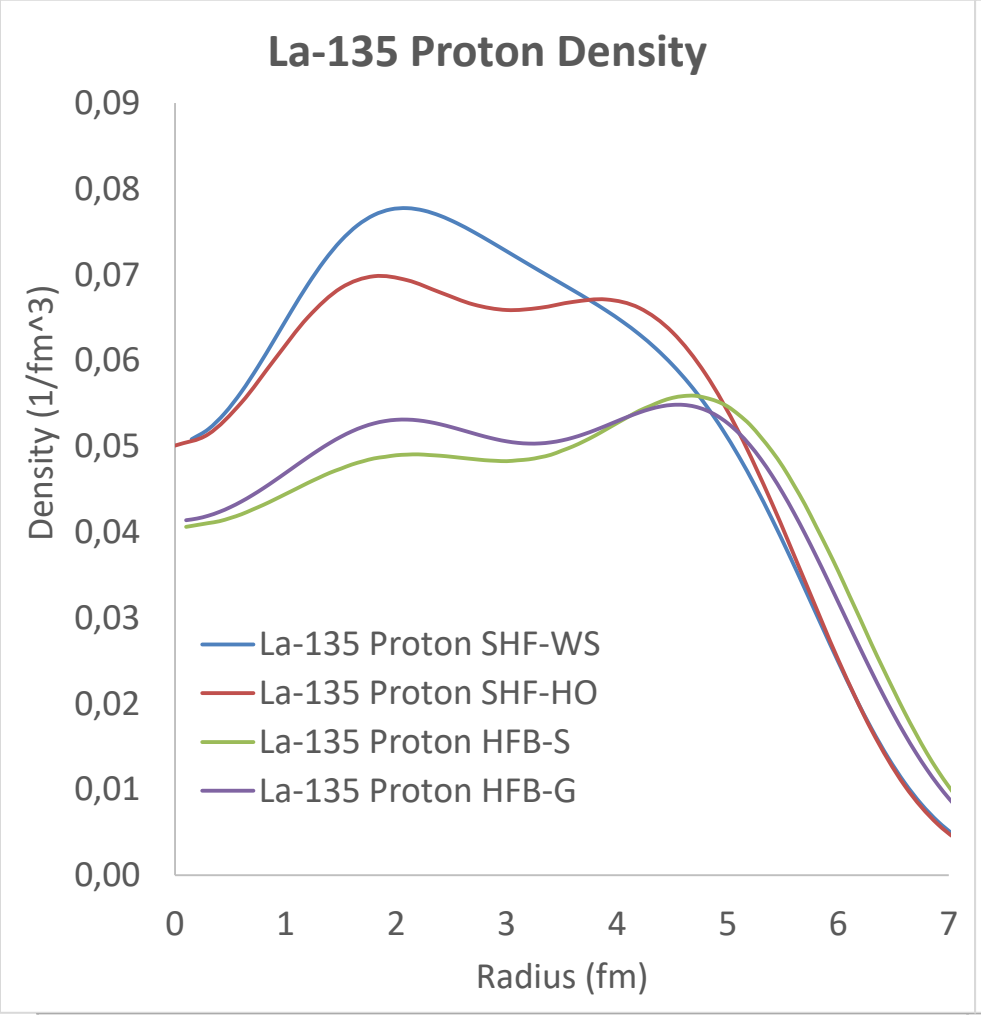
$$\left\langle r_c^2 \right\rangle = \left\langle r_p^2 \right\rangle + \left\langle r_p \right\rangle^2 \tag{8}$$

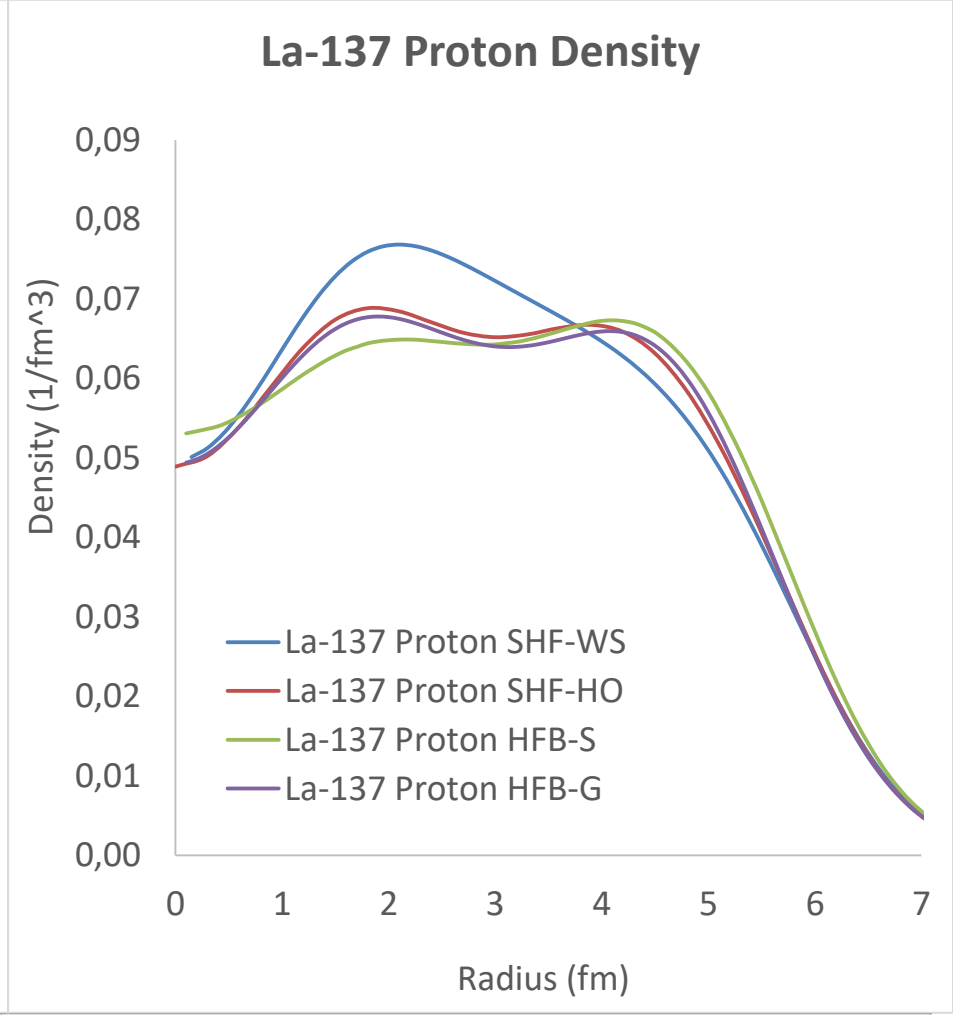
Skyrme Parameter Sets

Skyrme	t_0 (Mev.fm^3)	t_1 (Mev.fm^5)	t_2 (Mev.fm^5)	t_3 (MeV.fm$^{3(s+1)}$)	x_0	x_1	x_2	x_3	b_4 (MeV.fm^5)	b_{4p} (MeV.fm^5)	W_0 (MeV.fm^5)	alpha
SKa	-1602.78	570.88	-67.7	8000.0	-0.02	0	0	-0.286	62.5	62.5	125.0	1/3
SGII	-2645.0	340.0	-41.9	15595.0	0.09	-0.0588	1.423	0.06044	52.5	52.5	105.0	1/6
SkM	-2645.0	385.0	-120.0	15595.0	0.09	0	0	0	65.0	65.0	130.0	1/6
SLy4	-2488.913	486.818	-546.395	13777.0	0.834	-0.3438	-1.0	1.354	61.5	61.5	123.0	1/6
SLy5	-2483.45	484.23	-556.69	13757.0	0.776	-0.317	-1.0	1.263	62.5	62.5	125.0	1/6
SLy6	-2479.5	462.18	-448.61	13673.0	0.825	-0.465	-1.0	1.355	61.0	61.0	122.0	1/6
SLy7	-2480.8	461.29	-433.93	13669.0	0.848	-0.492	-1.0	1.393	62.5	62.5	125.0	1/6
SLy8	-2481.41	480.78	-538.34	13731.0	0.8024	-0.3424	-1.0	1.3061	62.8	59.625	122.425	1/6
SLy9	-2511.13	510.6	-429.8	13716.0	0.7998	-0.6213	-1.0	1.3727	55.0	64.0	119.0	1/6
SLy10	-2506.77	430.98	-304.95	13826.41	1.0398	-0.6745	-1.0	1.6833	37.93	52.755	90.685	1/6

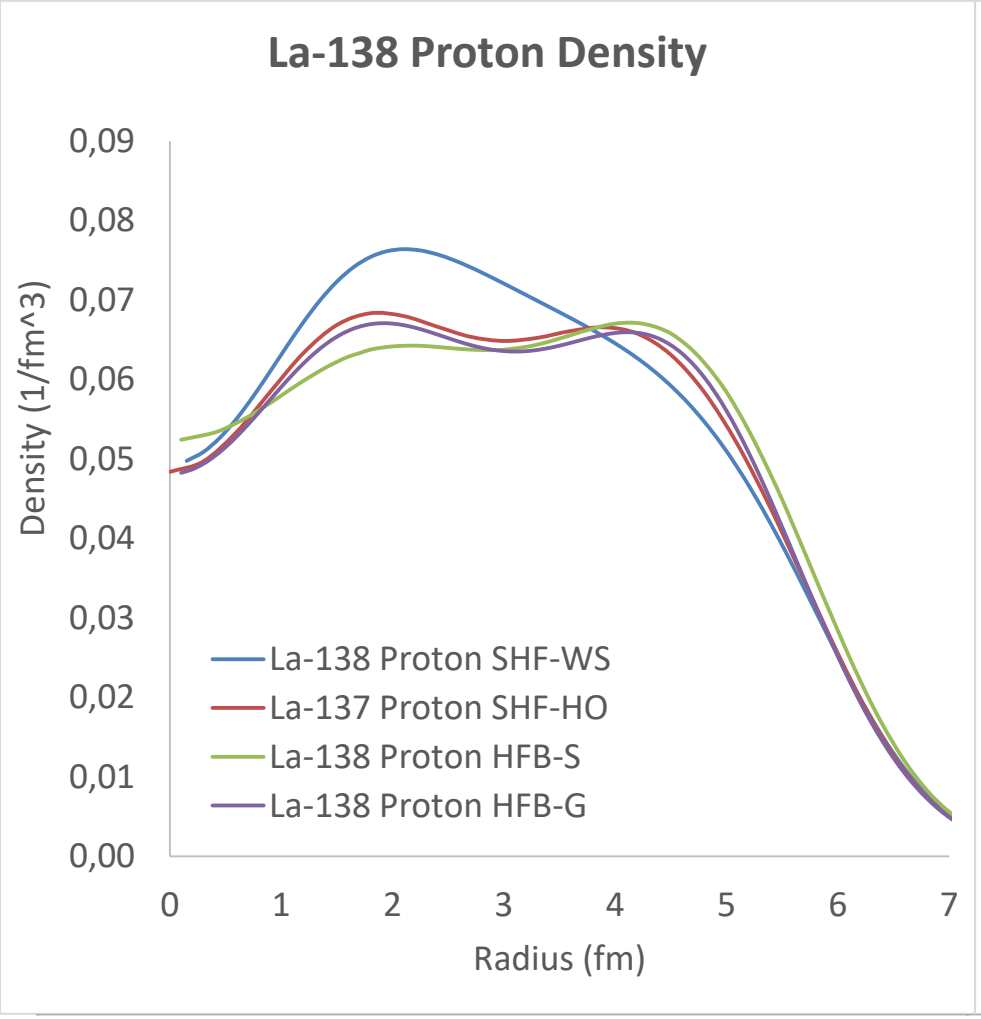
	Woods-Saxon				Harmonic Oscilator			
	Z=57 N=78 ^{135}La	Z=57 N=80 ^{137}La	Z=57 N=81 ^{138}La	Z=57 N=82 ^{139}La	Z=57 N=78 ^{135}La	Z=57 N=80 ^{137}La	Z=57 N=81 ^{138}La	Z=57 N=82 ^{139}La
R_{Charge} (EXP)	4.8488	4.8496	4.8473	4.8550	4.8488	4.8496	4.8473	4.8550
R_{Charge} (SkA)	4.7786	4.8530	4.8574	4.8618	4.9565	4.9674	4.9592	4.9636
R_{Charge} (GS6)	4.4402	4.4493	4.4538		4.4453	4.4604	4.4706	4.4788
R_{Charge} (SkM*)	4.8270	4.8348	4.8387	4.8428	4.8539	4.8625	4.8666	4.8710
R_{Charge} (SGII)	4.8226	4.8318	4.8364	4.8411	4.8489	4.8588	4.8634	4.8685
R_{Charge} (SLy4)	4.8371	4.8457	4.8500	4.8544	4.8591	4.8673	4.8714	4.8759
R_{Charge} (SLy5)	4.8273	4.8357	4.8399	4.8441	4.8491	4.8573	4.8615	4.8657
R_{Charge} (SLy6)	4.8257	4.8346	4.8390	4.8435	4.8492	4.8582	4.8627	4.8674
R_{Charge} (SLy7)	4.8273	4.8361	4.8405	4.8450	4.8514	4.8604	4.8650	4.8696
R_{Charge} (SLy8)	4.8285	4.8371	4.8414	4.8457	4.8505	4.8589	4.8631	4.8675
R_{Charge} (SLy9)	4.9060	4.9157	4.9206	4.9255	4.9285	4.9383	4.9436	4.9488
R_{Charge} (SLy10)	4.8517	4.8626	4.8680	4.8735	4.8698	4.8815	4.8873	4.8930

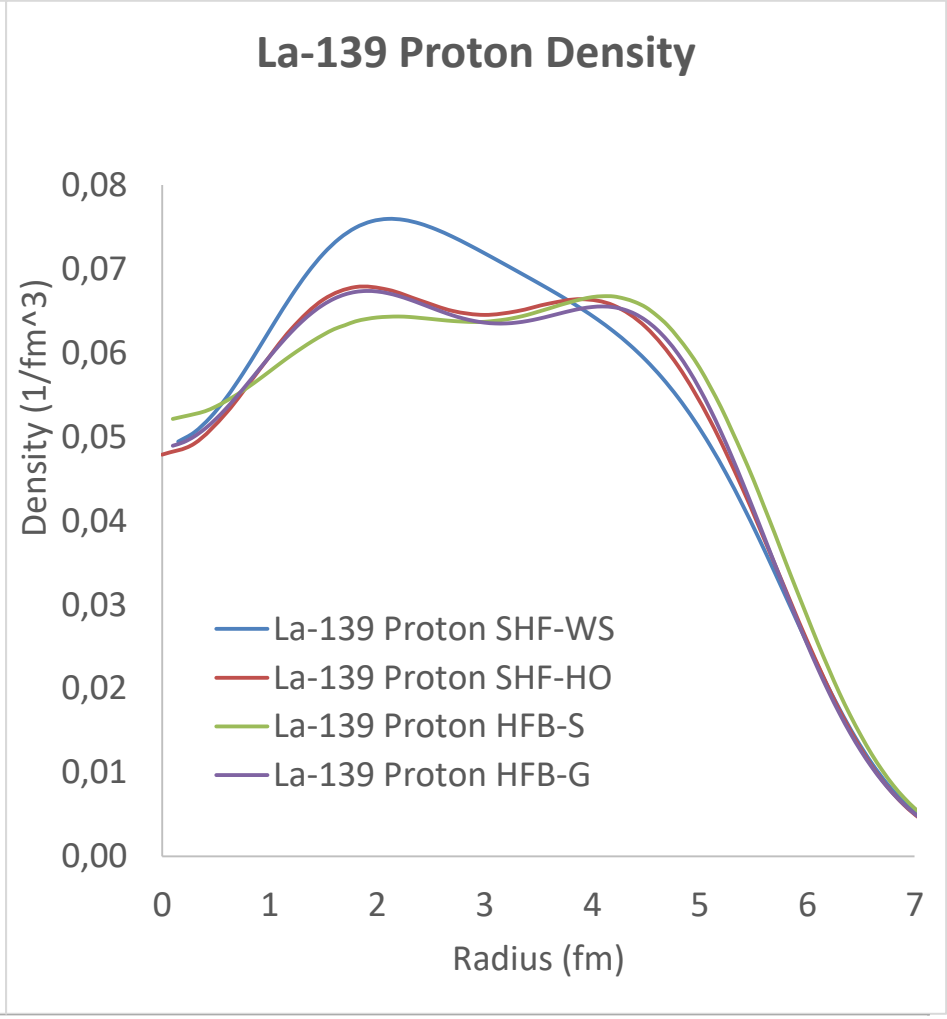
Proton Densities



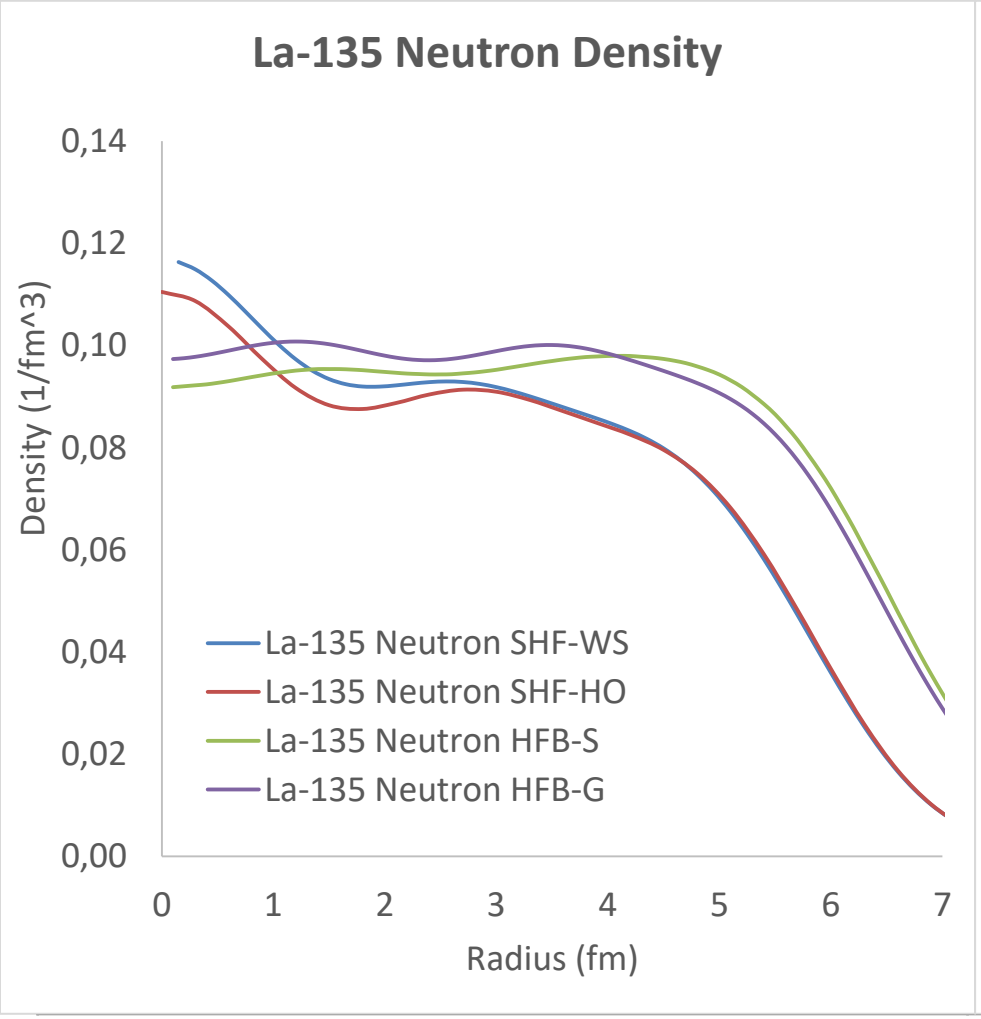


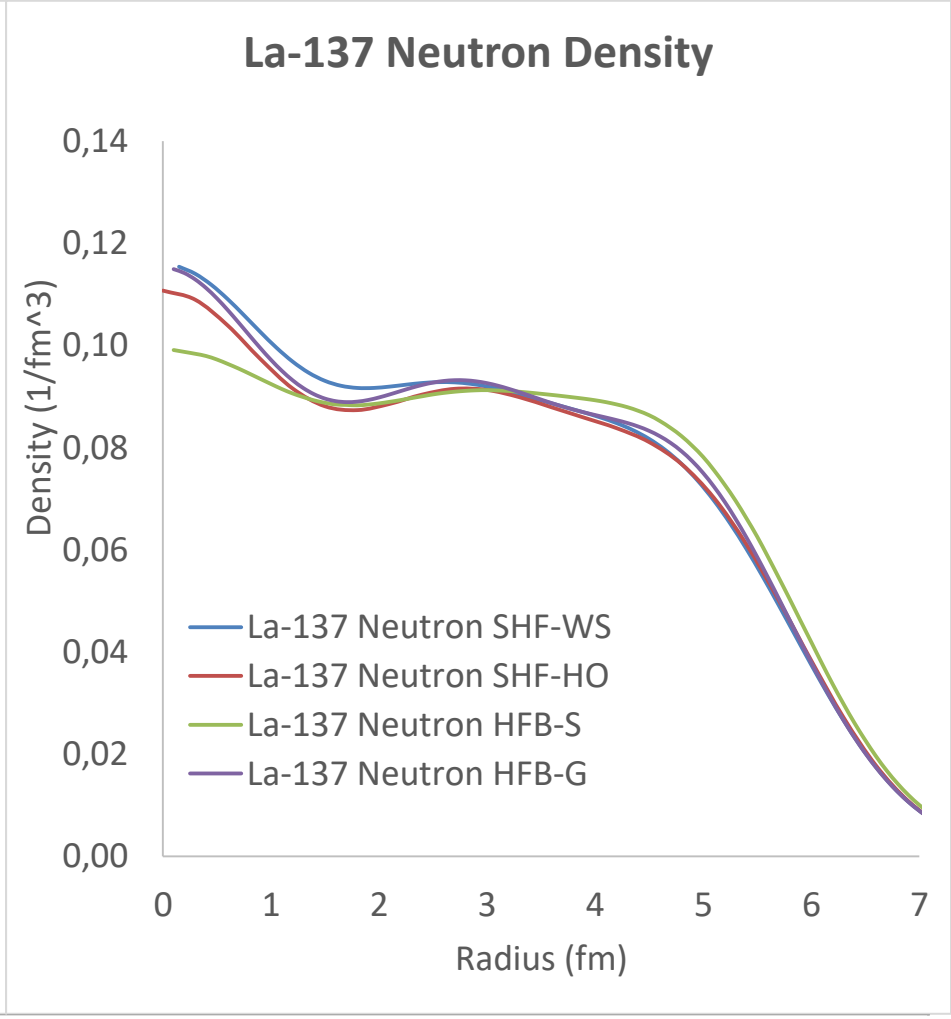
Proton Densities





Neutron Densities





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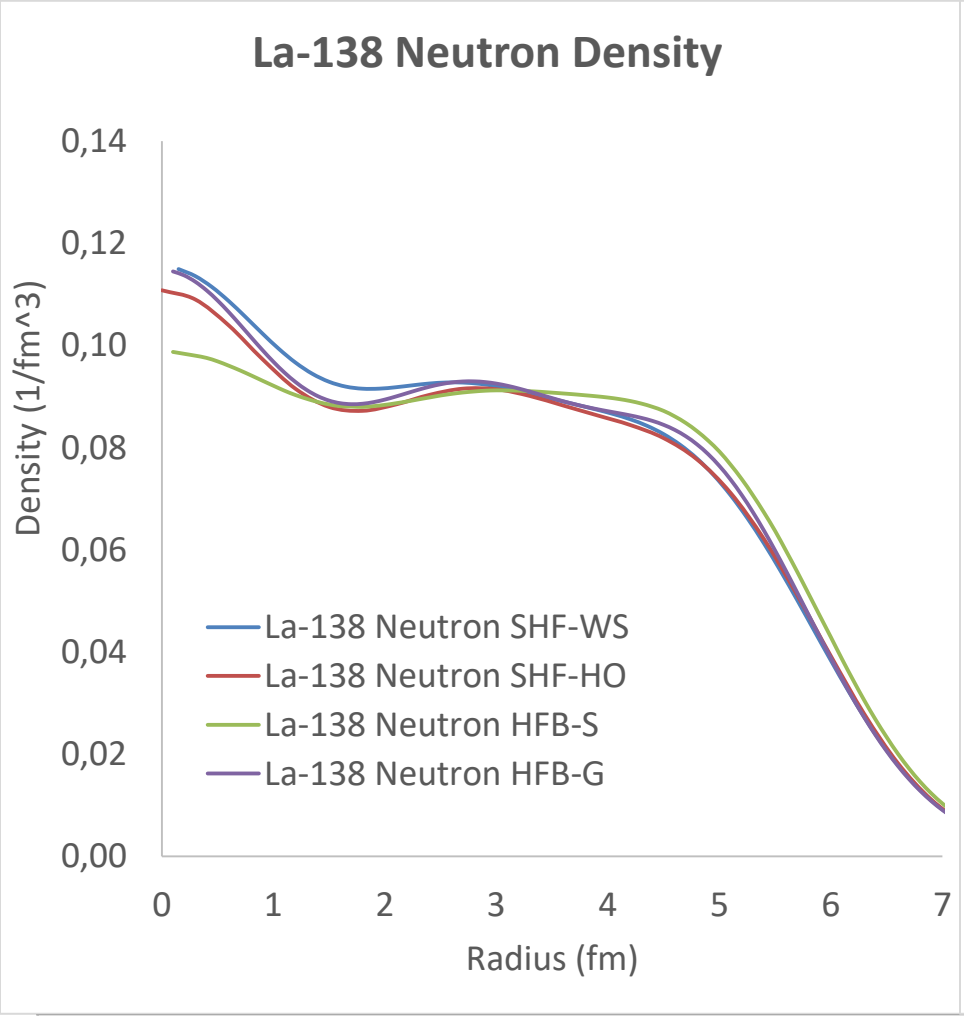

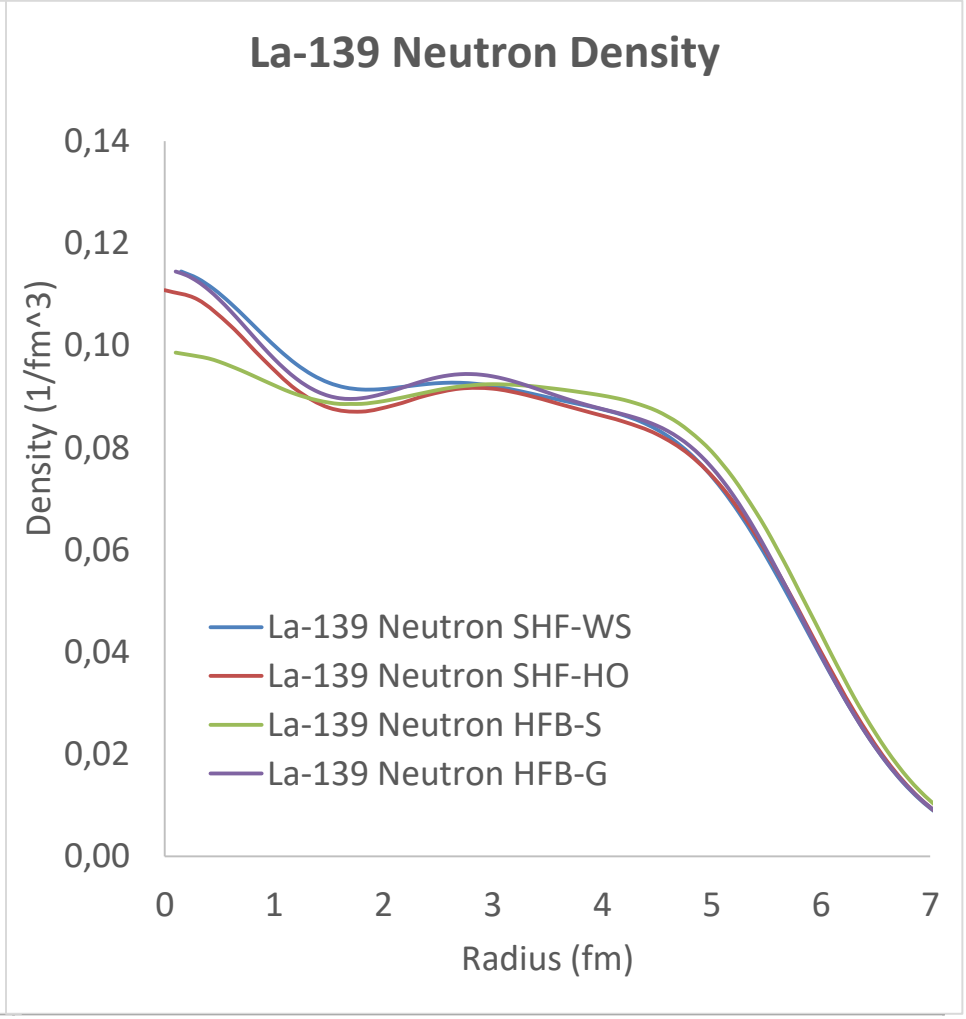

Thanks for your attention.