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Collective states of even-even nuclei in γ-rigid quadrupole Hamiltonian with Minimal Length under the sextic potential

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Summary

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the main aims

- we study the collective states of even-even nuclei in γ-rigid mode within the sextic potential and the Minimal Length (ML) formalism in Bohr–Mottelson model. The eigenvalues problem for this latter is solved by means conjointly of Quasi-Exact Solvability (QES) and a Quantum Perturbation Method (QPM).
- We study the effect of ML on the energy spectra,on the transition rates,on the shape phase transition within an isotopic chain and on the moments of inertia.
- The model is conventionally called X(3)-SML.

The Bohr-Mottelson Hamiltonian [1, 2] :

$$H_B = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{k=1,2,3}\frac{Q_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)}\right] + V(\beta,\gamma), \quad (1)$$

The Minimal Length or Generalized Uncertainty Principle (GUP)

$$[\hat{X}, \hat{P}] = i\hbar\left(1 + \alpha^2\hat{P}^2\right), \quad (2)$$

Where α represents the ML parameter (is very small positive parameter), this commutation relation leads to the uncertainty relation

$$(\Delta X)(\Delta P) \geq \frac{\hbar}{2}\left(1 + \alpha(\Delta P)^2 + \tau\right), \tau = \alpha\langle\widehat{P}\rangle^2 \quad (3)$$

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The application of ML to $\gamma-$rigid

$$\left[-\frac{\hbar^2}{2B}\Delta+\frac{\alpha\hbar^4}{2B}\Delta^2+V(\beta)-E\right]\psi(\beta,\theta,\phi)=0. \qquad (4)$$

with

$$\Delta=\frac{1}{\beta^2}\frac{\partial}{\partial\beta}\beta^2\frac{\partial}{\partial\beta}-\frac{\Delta_\Omega}{\beta^2}, \qquad (5)$$

where Δ_Ω is the angular part of the Laplace operator checking

$$\Delta_\Omega Y_{LM}(\theta,\phi)=-\frac{L(L+1)}{3}Y_{LM}(\theta,\phi), \qquad (6)$$

By considering the auxiliary wave function as in [3]

$$\psi(\beta,\theta,\phi)=\left[1+2\alpha\hbar^2\Delta\right]F(\beta)Y_{LM}(\theta,\phi), \qquad (7)$$

which leads to the following

$$\left[\frac{1}{\beta^2}\frac{d}{d\beta}\beta^2\frac{d}{d\beta}-\frac{L(L+1)}{3\beta^2}+\frac{2B}{\hbar^2}\left(\frac{E-V(\beta)}{1+4B\alpha(E-V(\beta))}\right)\right]F(\beta)=0. \qquad (8)$$

The expansion of the second term in power series of α

$$(1+4B\alpha(E-V(\beta)))^{-1} \simeq 1-4B\alpha(E-V(\beta)), \tag{9}$$

The radial part becomes :

$$\left[H_{\beta}^{(0)}+H_{\beta}^{(p)}\right]F(\beta)=EF(\beta), \tag{10}$$

where

$$\begin{cases} H_{\beta}^{(0)} = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^2}\frac{d}{d\beta}\beta^2\frac{d}{d\beta}-\frac{L(L+1)}{3\beta^2}-V(\beta)\right] \\ H_{\beta}^{(p)} = 4B\alpha(E-V(\beta))^2 \end{cases} \tag{11}$$

Solutions of β-equation for $\alpha = 0$ within QES

The Schrödinger picture of the radial equation. This is realized easily by changing the unperturbed wave function $F^{(0)}(\beta)$ by $\frac{\xi(\beta)}{\beta}$:

$$\left[-\frac{d^2}{d\beta^2} + \frac{L(L+1)}{3\beta^2} + v(\beta)\right]\xi(\beta) = \varepsilon\xi(\beta), \varepsilon = \frac{2B}{\hbar^2}E, v(\beta) = \frac{2B}{\hbar^2}V(\beta) \quad (12)$$

$$v(\beta) = \left[b^2 - 4a(s + \frac{1}{2} + M')\right]\beta^2 + 2ab\beta^4 + a^2\beta^6 \quad (13)$$

$$\left(2s - \frac{1}{2}\right)(2s - \frac{3}{2}) = \frac{L(L+1)}{3}. \equiv s + \frac{1}{2} + M' = const = c. \quad (14)$$

$$(M', L) : (k, 0), (k-1, 4), (k-2, 8).... \Rightarrow k + \tfrac{5}{4} = c_0^{(k)}, \quad (15)$$

$$(M', L) : (k, 2), (k-1, 6), (k-2, 10).... \Rightarrow k + \tfrac{7}{4} = c_2^{(k)}. \quad (16)$$

Solutions of β-equation for $\alpha = 0$ within QES

The sextic potential equation (12) can be more simplified by reducing the number of parameters through the change of variable $\beta = ya^{-1/4}$ and adopting the notations $\varrho = b/\sqrt{a}$ and $\varepsilon_y = \varepsilon/\sqrt{a}$:

$$\left[-\frac{d^2}{dy^2} + \frac{L(L+1)}{3y^2} + v_m^{(k)}(y)\right]\eta^{(0)}(y) = \varepsilon_y \eta^{(0)}(y) \qquad (17)$$

where,

$$v_m^{(k)}(y) = (\varrho^2 - 4c_m^{(k)})y^2 + 2\varrho y^4 + y^6 + u_m^{(k)}(\varrho) \;\; m = 0, 2. \qquad (18)$$

by considering the ansatz function :

$$\eta^{(0)}(y) = \eta^{(0)}_{n,L}(y) = N_{n,L} P^{(M')}_{n,L}\left(y^2\right) y^{\frac{L}{2}+1} e^{-\frac{y^4}{4} - \frac{\varrho y^2}{2}}, n = 0, 1, 2, \qquad (19)$$

Solutions of β-equation for $\alpha = 0$ within QES

Eq. (17) with potential (18) is reduced to the equation

$$\left[-\left(\frac{d^2}{dy^2}+\frac{4s'-1}{y}\frac{d}{dy}\right)+2\varrho\frac{d}{dy}\right]P_{n,L}^{(M')}\left(y^2\right) + 2y^2\left(y\frac{d}{dy}-2M'\right)P_{n,L}^{(M')}\left(y^2\right)=\lambda P_{n,L}^{(M')}\left(y^2\right) \quad (20)$$

$$\lambda=\lambda_{n,L}^{(k)}(\varrho)=\varepsilon_y-4bs'-u_m^{(k)}(\varrho)-\frac{1}{\langle y^2\rangle_{n,L}}\frac{L}{6}\left(\frac{L}{2}-1\right), \quad (21)$$

Finally,

$$E_{n,L}^{(0)}=E=\frac{\hbar^2\sqrt{a}}{2B}\left[\lambda_{n,L}^{(k)}(\varrho)+\varrho(L+3)+u_m^{(k)}(\varrho)+\frac{1}{\langle y^2\rangle_{n,L}}\frac{L}{6}\left(\frac{L}{2}-1\right)\right]. \quad (22)$$

Correction to the energy spectrum of system by QPM

In the perturbation theory

$$E_{n,L}^{Corr} = E_{n,L}^{(0)} + \Delta E_{n,L}, \tag{23}$$

where

$$\Delta E_{n,L} = \kappa\sqrt{a}\frac{\hbar^4}{B}\Big[\overline{y^{12}}+4\varrho\overline{y^{10}}+2\left(2\varrho^2 + A_m^{(k)}\right)\overline{y^8}+2\left(2\varrho^2 A_m^{(k)} - \varsigma_{n,L}^{(0)}\right)\overline{y^6}$$
$$+\left((A_m^{(k)})^2 - 4\varrho\varsigma_{n,L}^{(0)}\right)\overline{y^4} - 2A_m^{(k)}\varsigma_{n,L}^{(0)}\overline{y^2} + \left(u_m^{(k)}(\varrho) - \varsigma_{n,L}^{(0)}\right)\Big], \tag{24}$$

where $A_m^{(k)} = \left(\varrho^2 - 4\,c_m^{(k)}\right)$,
$\varsigma_{n,L}^{(0)} = \lambda_{n,L}^{(k)}(\varrho) + \varrho(L+3) + \frac{1}{\langle y^2\rangle_{n,L}}\frac{L}{6}\left(\frac{L}{2}-1\right)$, $\kappa = \sqrt{a}\alpha$ is the new ML parameter and $\overline{y^t}(t = 2, 4, 6, 8, 12)$ are the mean value of y^t

Correction to the wave function of system by QPM

$$\eta_{n,L}^{Corr}(y) = \eta_{n,L}^{(0)}(y) + \sum_{k\neq n}\left[\frac{\int_0^\infty \eta_{k,L}^{(0)}(y)\vartheta\left(n,\varrho,\kappa,\varsigma_{n,L}^{(0)},A_m^{(k)}\right)\eta_{n,L}^{(0)}(y)dy}{\varsigma_{n,L}^{(0)}-\varsigma_{k,L}^{(0)}}\right]\eta_{k,L}^{(0)}(y), \quad (25)$$

where

$$\vartheta\left(n,\varrho,\kappa,\varsigma_{n,L}^{(0)},A_m^{(k)}\right) = 2\kappa\hbar^2\Big[y^{12}+4\varrho y^{10}+2\left(2\varrho^2+A_m^{(k)}\right)y^8 + 2\left(2\varrho^2A_m^{(k)}-\varsigma_{n,L}^{(0)}\right)y^6+\left((A_m^{(k)})^2-4\varrho\varsigma_{n,L}^{(0)}\right)y^4-2A_m^{(k)}\varsigma_{n,L}^{(0)}y^2 + \left(u_m^{(k)}(\varrho)-\varsigma_{n,L}^{(0)}\right)\Big]. \quad (26)$$

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$R_{n,L}$ and $B(E_2)$

$$R_{n,L} = \frac{E^{Corr}_{n,L} - E^{Corr}_{0,0}}{E^{Corr}_{0,2} - E^{Corr}_{0,0}} \tag{27}$$

$$B(E_2) = T_{n,L,n',L'} = \frac{B\left(E2; n; L \to n', L'\right)}{B\left(E2; 0; 2 \to 0, 0\right)} = \left(\frac{C^{L2L'}_{0,0,0} I^{Corr}_{n,L,n',L'}}{C^{220}_{0,0,0} I^{Corr}_{0,2,0,0}}\right)^2, \tag{28}$$

where the radial matrix element $I^{Corr}_{n,L,n',L'}$ can be given either in β or y variable :

$$\begin{aligned} I^{Corr}_{n,L,n',L'} = & \int_0^{+\infty} F^{Corr}_{n,L}(\beta)\beta F^{Corr}_{n',L'}(\beta)\beta^2 \mathrm{d}\beta \\ & = a^{-1/4} \int_0^{+\infty} \eta^{Corr}_{n,L}(y) y \eta^{Corr}_{n',L'}(\beta)\mathrm{d}y. \end{aligned} \tag{29}$$

The sextic potential

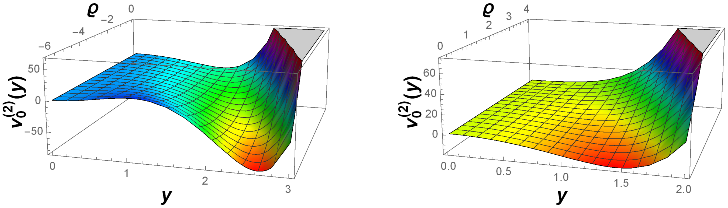
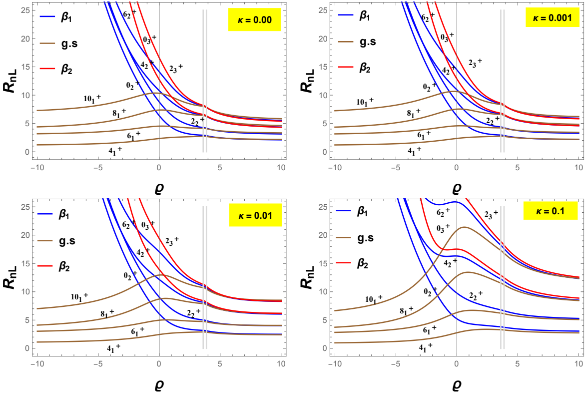
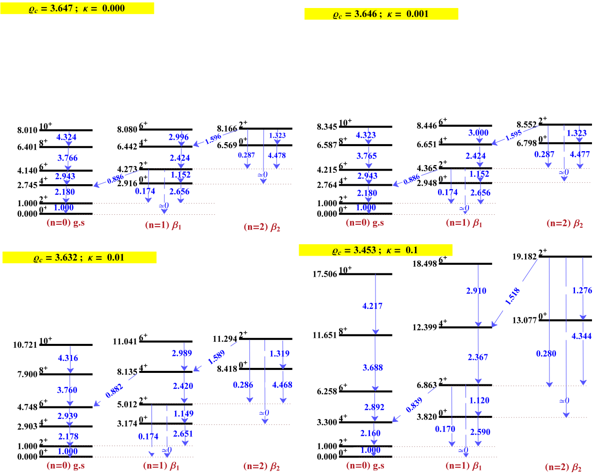


FIGURE: The shape evolution of the sextic potential $v_0^{(2)}(y)$, given by (18), as a function of the parameter ϱ for $c_0^{(2)} = \frac{13}{4}$ and $u_0^{(2)} = 0$.

The energy spectra

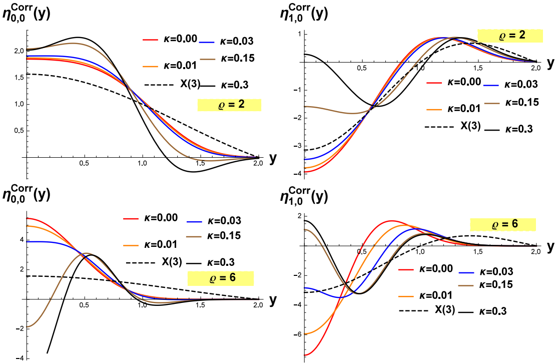


$R_{n,L}$ and $B(E_2)$ in ϱ_c



The ML eliminate partially this approximate degeneracy.

The corrected wave function



As a result the value ($\kappa = 0.03$) shows the limitation of the perturbation term.

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{116}Xe	Exp	2.33					
	X(3)	2.44	2.87	7.65	2.48	-	0.0
	X(3)-S	2.28	2.25	4.75	0.72	6.62	0.0
	X(3)-SML	2.20	2.16	4.69	0.68	18.30	0.0016
^{118}Xe	Exp	2.40	2.46	(5.10)			
	X(3)	2.44	2.87	7.65	0.99	-	0.0
	X(3)-S	2.61	2.71	6.01	0.49	4.16	0.0
	X(3)-SML	2.35	2.41	5.84	0.29	14.96	0.0070
^{120}Xe	Exp	2.47	2.82	(6.93)			
	X(3)	2.44	2.87	7.65	0.99	-	0.0
	X(3)-S	2.73	2.88	6.45	0.57	3.79	0.0
	X(3)-SML	2.40	2.50	6.22	0.36	14.89	0.0097
^{122}Xe	Exp	2.50	3.47	(7.63)			
	X(3)	2.44	2.87	7.65	0.36	-	0.0
	X(3)-S	2.63	3.29	7.72	0.30	2.11	0.0
	X(3)-SML	2.72	3.13	7.56	0.23	2.71	0.0020

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{124}Xe	Exp	2.48	3.58	(6.70)			
	X(3)	2.44	2.87	7.65	0.71	-	0.0
	X(3)-S	2.70	3.01	6.91	0.53	2.99	0.0
	X(3)-SML	2.44	2.57	6.54	0.49	13.03	0.0132
^{126}Xe	Exp	2.42	3.38	(6.57)			
	X(3)	2.44	2.87	7.65	0.85	-	0.0
	X(3)-S	2.72	2.86	6.40	0.55	3.83	0.0
	X(3)-SML	2.38	2.48	6.12	0.47	14.92	0.0090
^{128}Xe	Exp	2.33	3.57	(5.87)			
	X(3)	2.44	2.87	7.65	0.77	-	0.0
	X(3)-S	2.66	2.78	6.19	0.44	4.00	0.0
	X(3)-SML	2.66	2.78	6.19	0.44	4.00	0.0
^{130}Xe	Exp	2.25					
	X(3)	2.44	2.87	7.65	0.77	-	0.0
	X(3)-S	2.41	2.43	5.26	0.55	5.18	0.0
	X(3)-SML	2.41	2.43	5.26	0.55	5.18	0.0

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{100}Mo	Exp	2.12	1.30				
	X(3)	2.44	2.87	7.65	1.63	-	0.0
	X(3)-S	2.20	2.14	4.44	0.43	8.63	0.0
	X(3)-SML	2.15	2.10	4.43	0.39	38.20	0.0005
^{102}Mo	Exp	2.51	2.35				
	X(3)	2.44	2.87	7.65	0.56	-	0.0
	X(3)-S	2.67	3.09	7.16	0.63	2.65	0.0
	X(3)-SML	2.59	2.72	6.99	0.27	5.32	0.0104
^{98}Ru	Exp	2.14	2.03				
	X(3)	2.44	2.87	7.65	1.48	-	0.0
	X(3)-S	2.21	2.15	4.46	0.1	8.44	0.0
	X(3)-SML	2.17	2.12	4.47	0.04	16.00	0.0010
^{100}Ru	Exp	2.27	2.10				
	X(3)	2.44	2.87	7.65	0.65	-	0.0
	X(3)-S	2.58	2.66	5.89	0.37	4.28	0.0
	X(3)-SML	2.33	2.37	5.60	0.14	10.95	0.006

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{102}Ru	Exp	2.33	1.99				
	X(3)	2.44	2.87	7.65	0.86	-	0.0
	X(3)-S	2.50	2.55	5.59	0.28	4.63	0.0
	X(3)-SML	2.32	2.36	5.39	0.17	12.68	0.0057
^{104}Ru	Exp	2.48	(2.76)				
	X(3)	2.44	2.87	7.65	0.69	-	0.0
	X(3)-S	2.74	2.92	6.58	0.40	3.62	0.0
	X(3)-SML	2.53	2.62	6.47	0.25	5.42	0.0071
^{106}Ru	Exp	2.66	3.67				
	X(3)	2.44	2.87	7.65	0.83	-	0.0
	X(3)-S	2.55	4.10	9.36	0.34	1.16	0.0
	X(3)-SML	2.65	3.67	9.03	0.13	1.61	0.0027
^{108}Ru	Exp	2.75	4.03				
	X(3)	2.44	2.87	7.65	1.51	-	0.0
	X(3)-S	2.49	4.75	10.43	0.83	0.73	0.0
	X(3)-SML	2.67	4.01	10.29	0.16	1.27	0.0061

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{132}Ce	Exp	2.64	3.56				
	X(3)	2.44	2.87	7.65	0.79	-	0.0
	X(3)-S	2.65	3.18	7.43	0.70	2.36	0.0
	X(3)-SML	2.49	2.66	6.95	0.55	16.11	0.0173
^{134}Ce	Exp	2.56	3.75				
	X(3)	2.44	2.87	7.65	0.46	-	0.0
	X(3)-S	2.61	3.42	8.01	0.26	1.89	0.0
	X(3)-SML	2.65	3.32	7.95	0.24	2.10	0.0010
^{172}Os	Exp	2.66	3.33				
	X(3)	2.44	2.87	7.65	1.58	-	0.0
	X(3)-S	2.65	2.77	6.16	0.77	4.02	0.0
	X(3)-SML	2.35	2.41	5.78	0.70	11.17	0.0072
^{180}Pt	Exp	2.68	3.12	(7.69)			
	X(3)	2.44	2.87	7.65	1.03	-	0.0
	X(3)-S	2.56	3.92	9.03	0.69	1.31	0.0
	X(3)-SML	2.86	3.22	8.52	0.27	3.05	0.0085

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{182}Pt	Exp	2.71	3.22	(7.43)			
	X(3)	2.44	2.87	7.65	1.04	-	0.0
	X(3)-S	2.57	3.88	8.95	0.72	1.35	0.0
	X(3)-SML	2.90	3.17	8.37	0.28	3.67	0.0099
^{184}Pt	Exp	2.67	3.02				
	X(3)	2.44	2.87	7.65	0.83	-	0.0
	X(3)-S	2.57	3.79	8.77	0.65	1.44	0.0
	X(3)-SML	2.85	3.18	8.33	0.23	3.18	0.0078
^{186}Pt	Exp	2.56	2.46				
	X(3)	2.44	2.87	7.65	0.92	-	0.0
	X(3)-S	2.68	3.04	7.03	0.62	2.28	0.0
	X(3)-SML	2.56	2.69	6.88	0.40	5.59	0.0105
^{188}Pt	Exp	2.53	3.01				
	X(3)	2.44	2.87	7.65	0.36	-	0.0
	X(3)-S	2.65	3.17	7.39	0.38	2.40	0.0
	X(3)-SML	2.63	2.77	7.00	0.15	4.76	0.0082

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{194}Pt	Exp	2.47	3.23				
	X(3)	2.44	2.87	7.65	0.18	-	0.0
	X(3)-S	2.64	3.86	7.55	0.19	2.25	0.0
	X(3)-SML	2.67	3.13	7.30	0.16	2.53	0.0002
^{196}Pt	Exp	2.47	3.19				
	X(3)	2.44	2.87	7.65	0.44	-	0.0
	X(3)-S	2.72	2.96	6.76	0.33	3.25	0.0
	X(3)-SML	2.61	2.73	6.55	0.26	4.53	0.0043
^{146}Nd	Exp	2.30	2.02				
	X(3)	2.44	2.87	7.65	1.67	-	0.0
	X(3)-S	2.34	2.32	4.96	0.64	5.88	0.0
	X(3)-SML	2.24	2.22	4.93	0.57	13.12	0.0026
^{148}Nd	Exp	2.49	3.04	(5.30)			
	X(3)	2.44	2.87	7.65	1.40	-	0.0
	X(3)-S	2.61	2.71	6.02	0.39	4.15	0.0
	X(3)-SML	2.52	2.59	5.99	0.38	4.82	0.0025

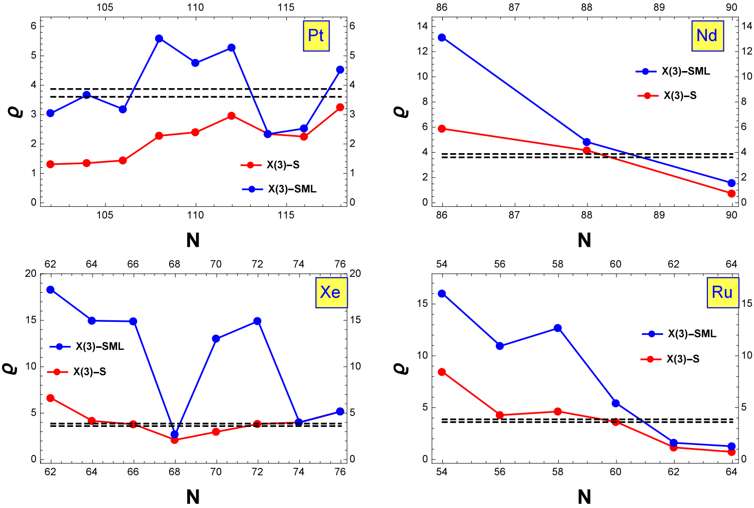
Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{150}Nd	Exp	2.93	5.19	(13.35)			
	X(3)	2.44	2.87	7.65	2.00	-	0.0
	X(3)-S	2.49	4.75	10.43	1.10	0.73	0.0
	X(3)-SML	2.73	3.78	10.00	0.80	1.56	0.0074
^{150}Sm	Exp	2.32	2.22	(3.76)			
	X(3)	2.44	2.87	7.65	1.98	-	0.0
	X(3)-S	2.36	2.36	5.07	0.41	5.58	0.0
	X(3)-SML	2.23	2.22	4.95	0.33	21.80	0.0021
^{152}Sm	Exp	3.01	5.62	8.89			
	X(3)	2.44	2.87	7.65	1.62	-	0.0
	X(3)-S	2.55	4.10	9.36	1.59	1.16	0.0
	X(3)-SML	2.95	3.24	8.91	1.26	3.70	0.0287

Energy Spectrum

Nucleus		$R_{0,4}$	$R_{1,0}$	$R_{2,0}$	σ	ϱ	κ
^{154}Gd	Exp	3.01	5.53	9.60			
	X(3)	2.44	2.87	7.65	1.51	-	0.0
	X(3)-S	2.53	4.33	9.75	1.51	0.99	0.0
	X(3)-SML	2.97	3.28	9.21	1.14	3.68	0.0165
^{156}Dy	Exp	2.93	4.90	10.00			
	X(3)	2.44	2.87	7.65	1.28	-	0.0
	X(3)-S	2.53	4.31	9.71	1.28	1.01	0.0
	X(3)-SML	2.96	3.26	9.04	0.99	3.65	0.0148

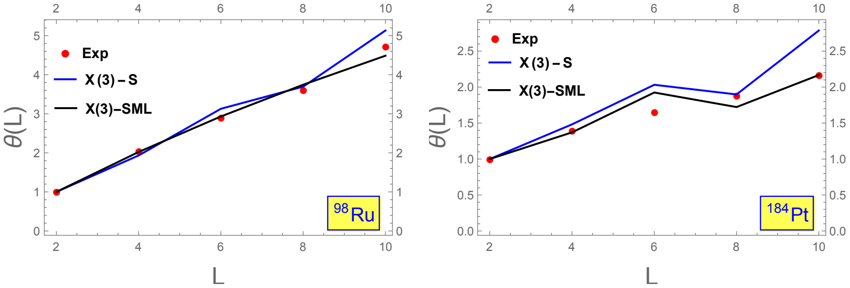
The ML effect on the shape phase transition



The moment of inertia of the ground state

The moment of inertia of the ground state [4] :

$$\theta(L) = \frac{R}{\omega} = \frac{1}{2}\frac{dE}{dR^2} \approx \frac{2L-1}{E(L)-E(L-2)}, R^2 = L(L+1). \quad (30)$$



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Conclusion

- The Bohr-Mottelson Hamiltonian in the γ-rigid regime within the Minimal Length (ML) formalism with sextic potential have been solved conjointly by mens of QES and QPM .
- The new elaborated model has allowed us to reproduce well the experimental data for energy ratios, transition rates and moments of inertia.
- the ML removed partially the degeneracy of states with different angular momenta $\Delta L = 2$ belonging to different bands in the critical point.
- The ML changed dramatically the shape phase transition within an isotopic chain.

Perspectives

- consider the quasi-exact solvability orders $k > 2$ for $\mathrm{X}(3)$-Sextic in the presence of ML and to verify whether the theoretical results will be improved more than in the case of k=2.
- correct the energies and the wave functions to the second order instead of the first one, and to study its effect on the energy ratios and transition rates.

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