**PAKİSTAN-TÜRKİYE KARDEŞ!!!**

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**HOŞ BULDUM..**

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# Spectroscopic Study of $^{9}$Be ($p$, $\gamma$) $^{10}$B at Astrophysical Conditions


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# Outlines

1. Prelude
2. Background
3. Objectives
4. Motivation
5. Formalism
6. Results and Discussion
7. Conclusion

# Prelude

- Capture rates are quantities of crucial importance in nuclear astrophysics and considerable attempts have been devoted in the last decades to calculate them.
- The $^{9}$Be ($p$, $\gamma$) $^{10}$B reaction is important for the determination of Lithium, Beryllium and Boron abundances and spectroscopic study of Boron.
- The reaction $^{9}$Be ($p$, $\gamma$) $^{10}$B is investigated within the energy range (0--0.6) MeV using the potential model.
- The total cross-section is taken as the sum of direct and resonance transitions.
- Based on the computed cross-section we calculate the capture rates within the selected range of temperature $T_9$ = (0.006 – 1)*. Our model-based capture rates and cross-sections show a satisfactory agreement with the previously published results, specially towards high $T_9$ values.

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*** $T_9 = 10^9$ Kelvin**

# Background

❑There are at least three nucleosynthesis sites

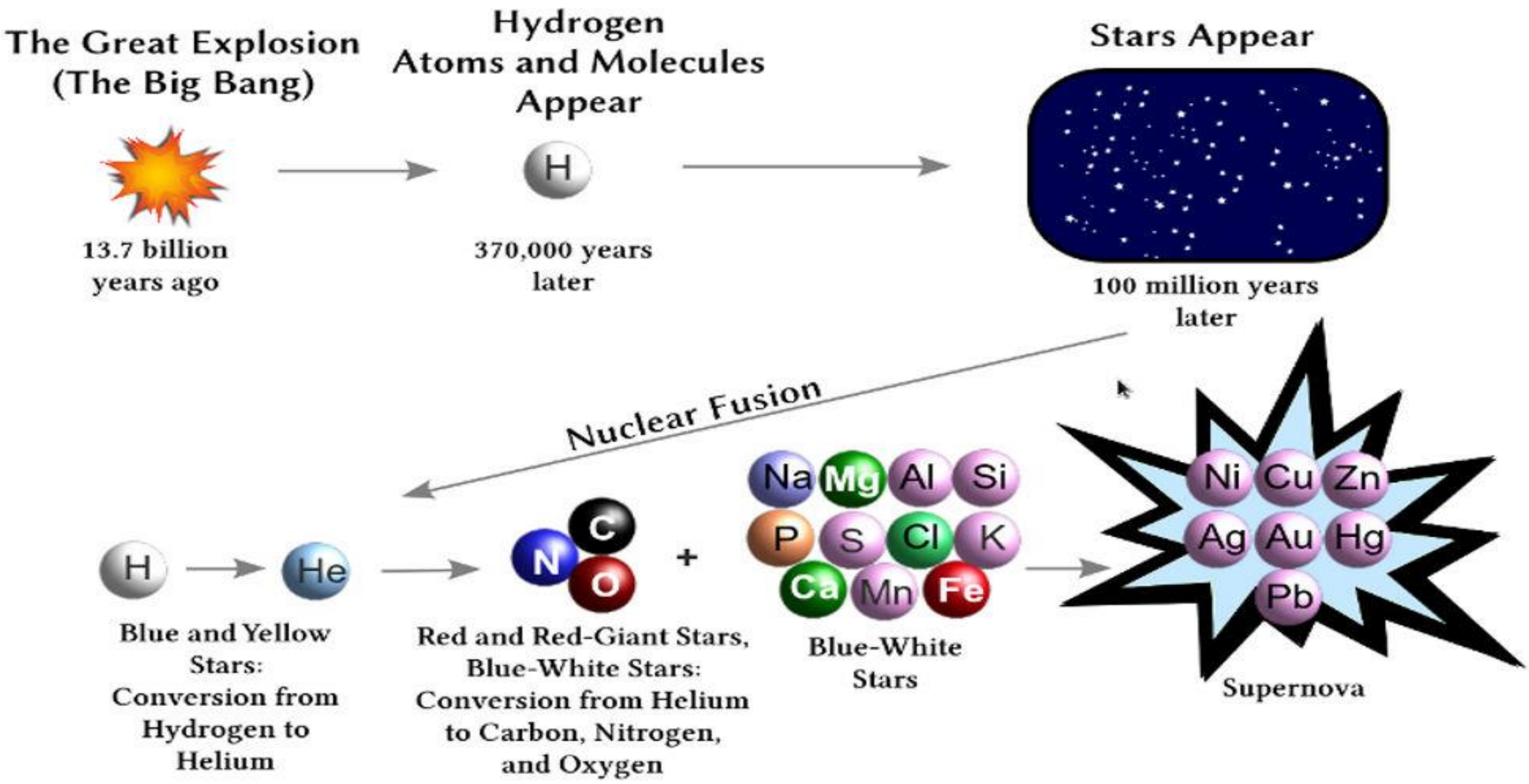
✓The Big Bang where Hydrogen and Helium were produced.

✓Stars where all elements from Carbon to Uranium are synthesized.

✓Interstellar medium in galaxies where Lithium (part of), Beryllium and Boron are made by non-thermal processes.

# Background

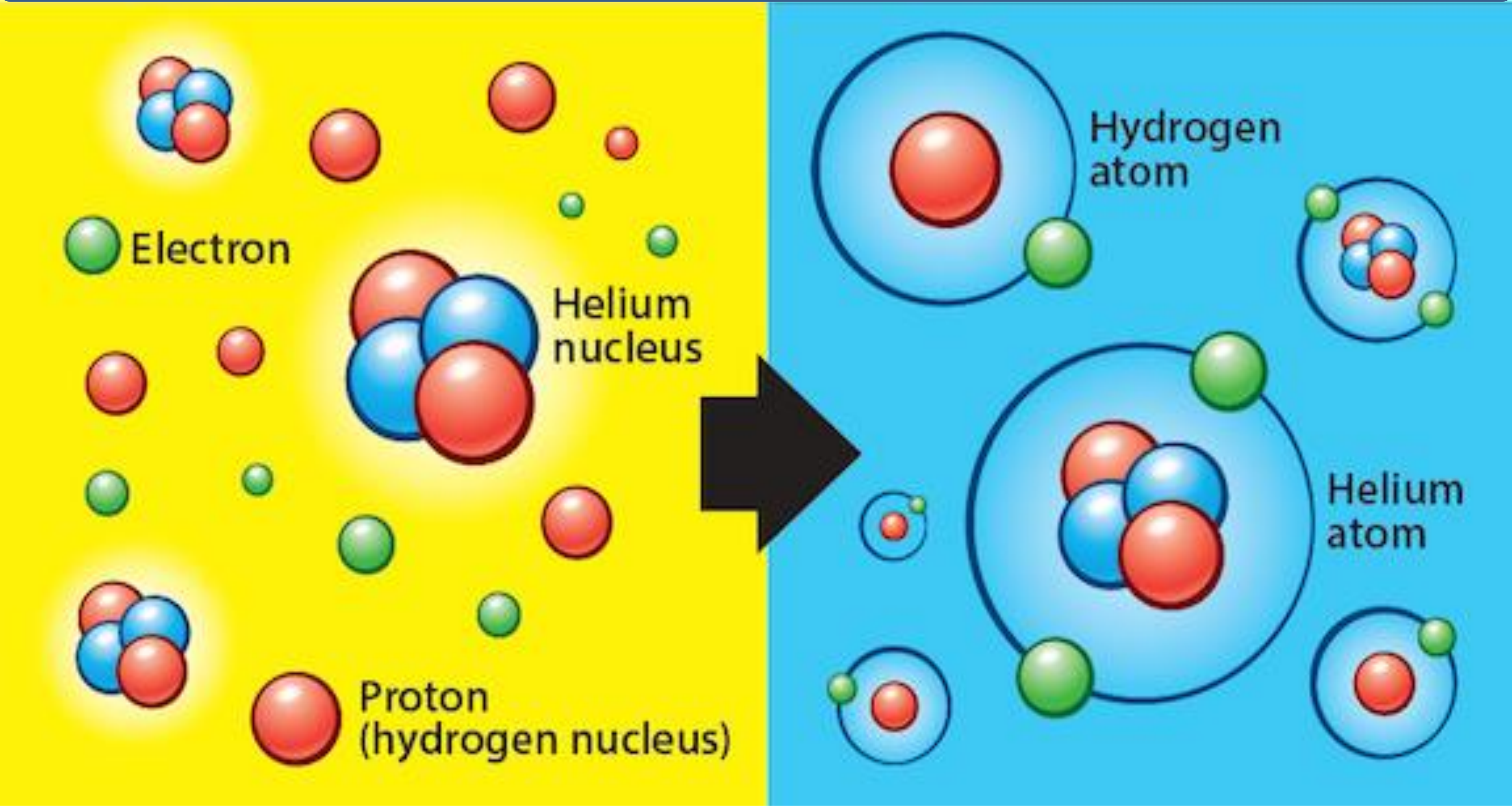
## Elements, made in the early Universe




https://www.pinterest.com/pin/200339883396798446/

# Background

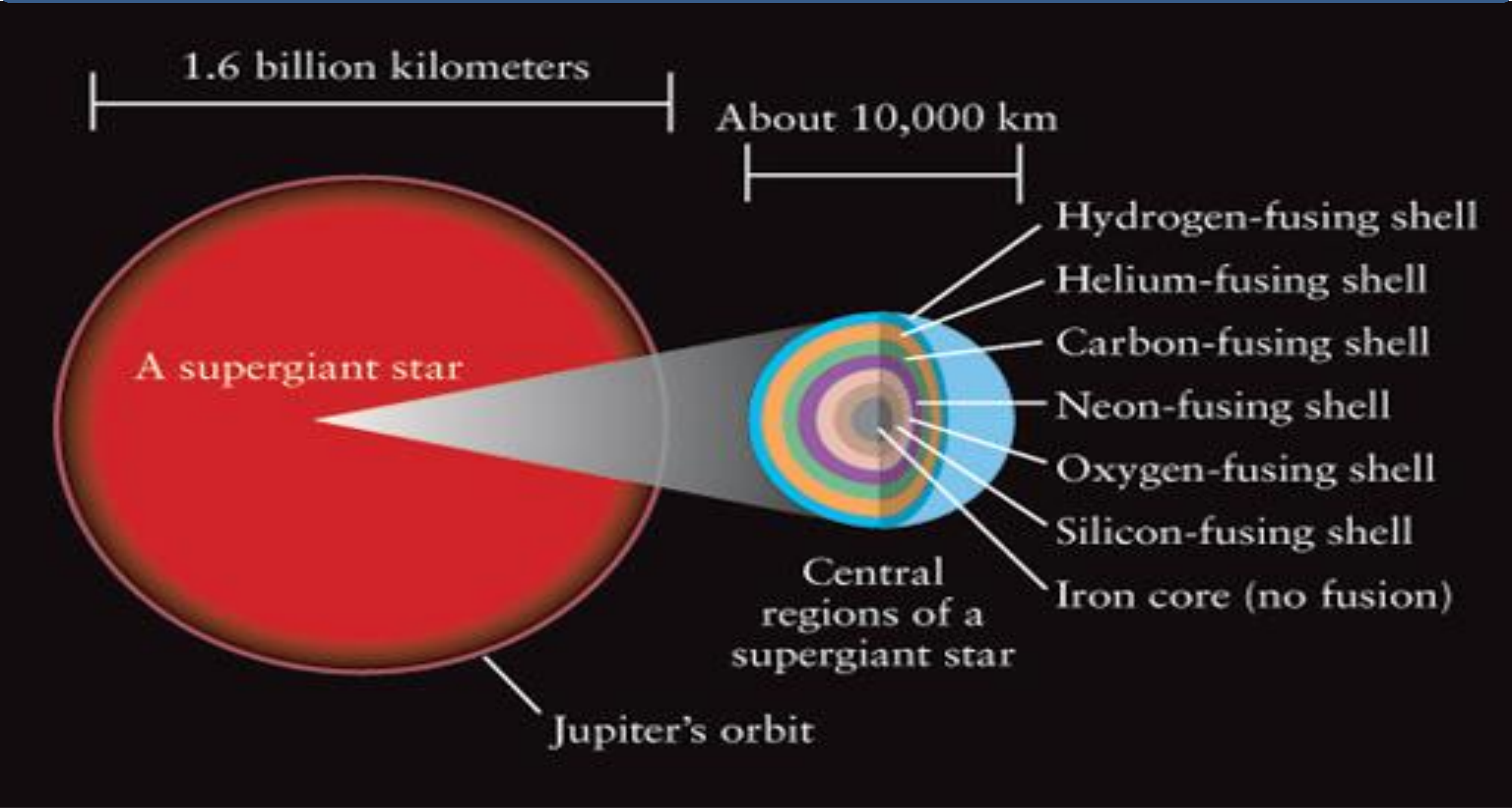
## Elements, made in the early Universe




https://astronomy.com/magazine/ask-astro/2018/12/the-first-element

# Background

## Elements, all made in stars

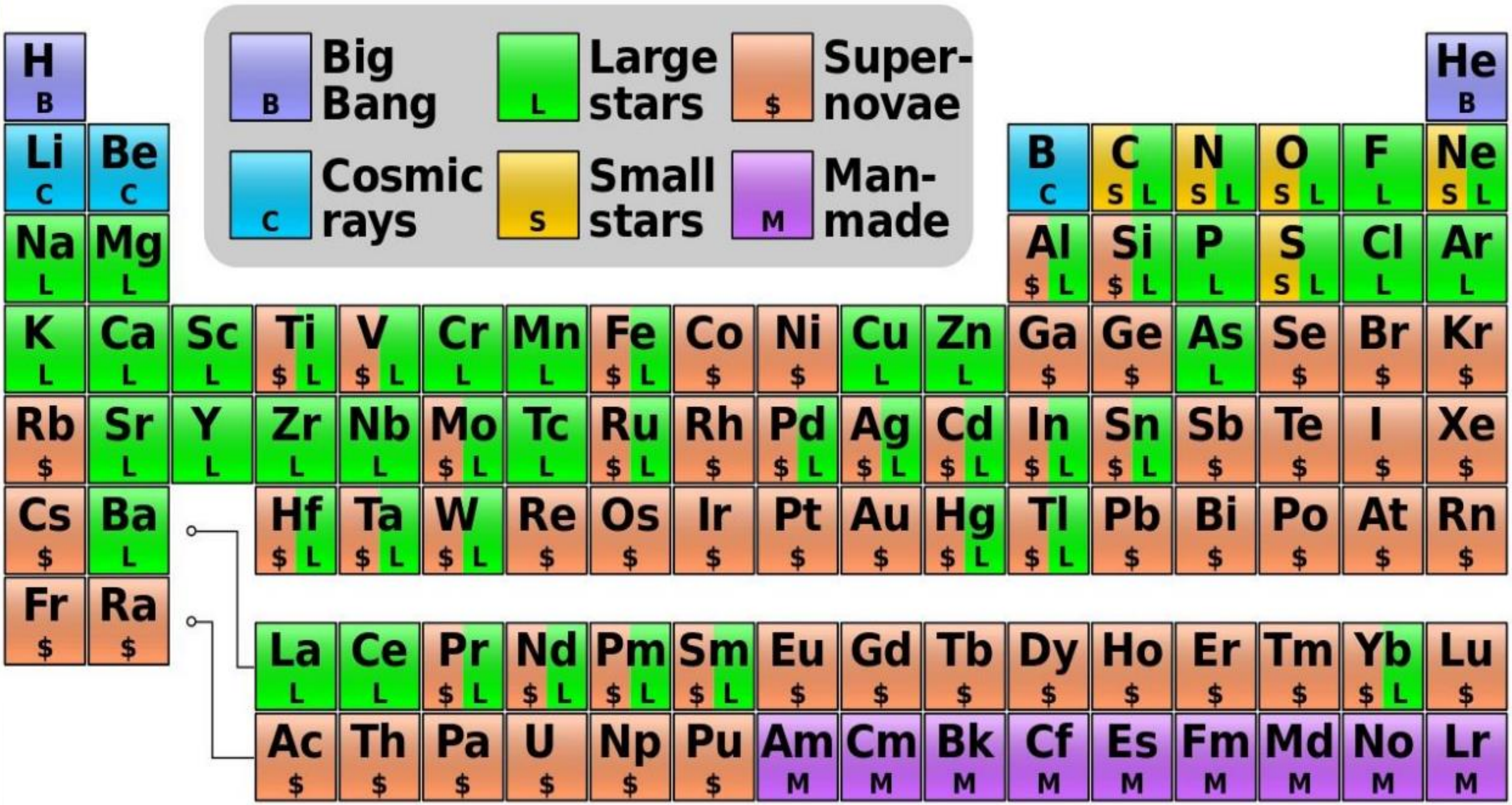



**https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect18/lecture18.html**

# Background

# Background

## Elemental classification




https://www.zmescience.com/space/where-elements-come-from-053543/

# Background

## Lithium-Beryllium-Boron (LiBeB): Origin and Evolution

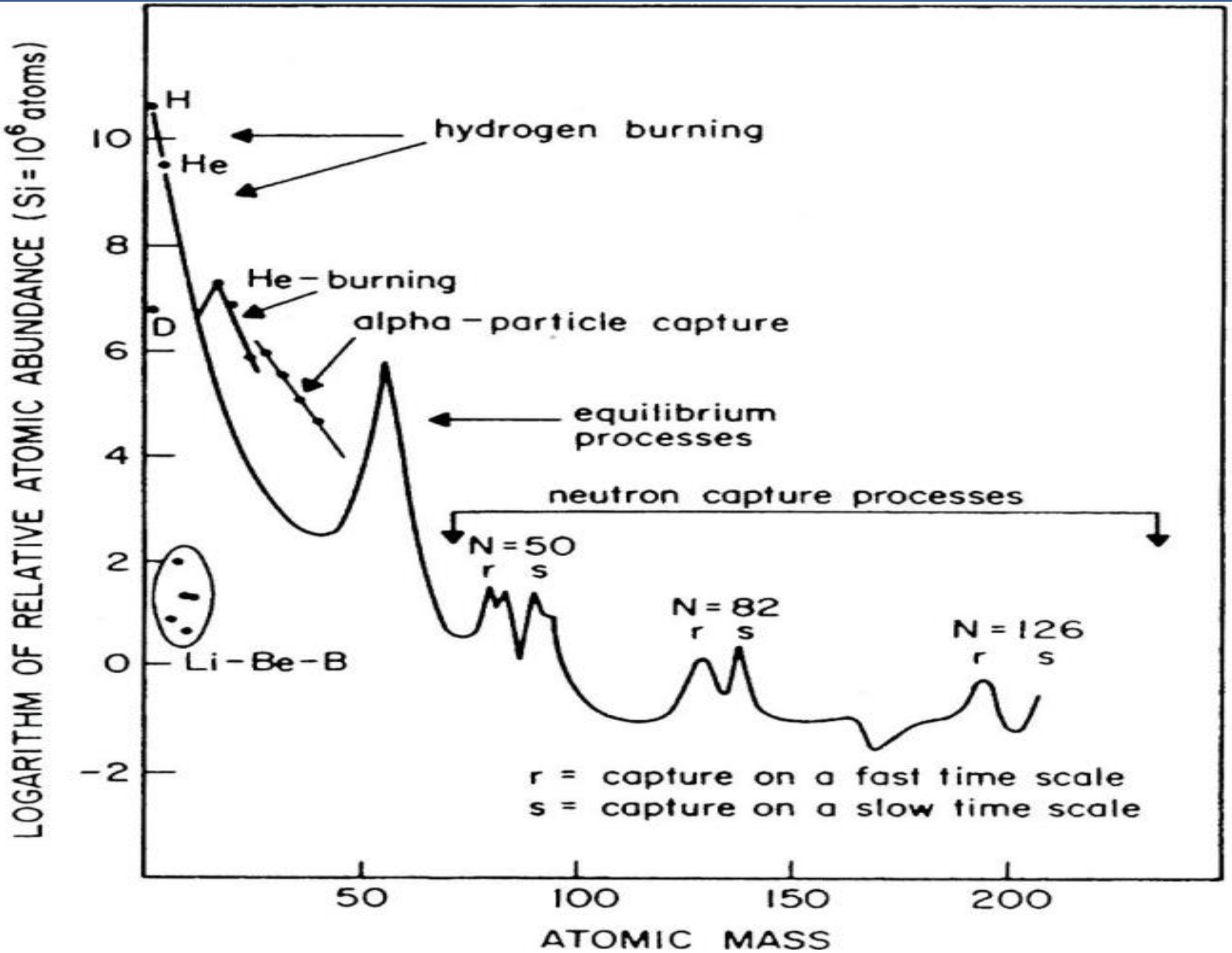
The origin and evolution of Lithium-Beryllium-Boron (LiBeB) is a crossing point between different astrophysical fields

- Galactic evolution and non-thermal nucleosynthesis
- Stellar nucleosynthesis

Light element nucleosynthesis is important in nuclear astrophysics. Specifically, the fragile nuclei, LiBeB are not generated in the normal course of stellar nucleosynthesis. Except $^{7}$Li, LiBeB is destroyed in stellar interiors. This characteristic is reflected in the low abundance of these nuclei.

# Background

## (LiBeB) Abundance




***C. Iliadis, Nuclear physics of stars. John Wiley & Sons, 2015.***

# Background

## Lithium-Beryllium-Boron (LiBeB): Origin and Evolution

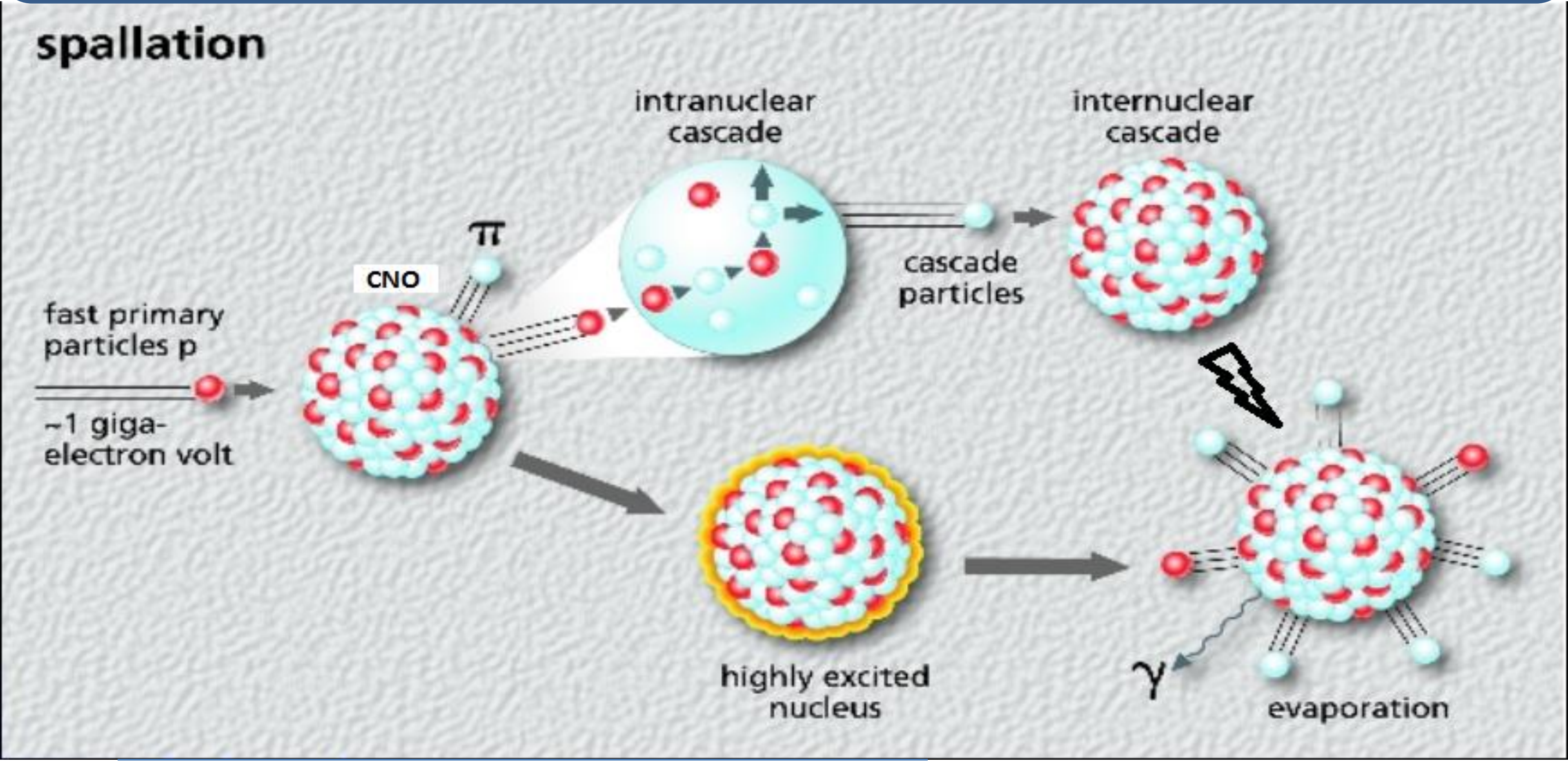
The isotopes of Lithium, $^{6}$Li and $^{7}$Li, have a special nucleosynthetic status. However, their origins are quite different. The story of $^{7}$Li is perhaps better-known, as this nuclide dominates by far the Li production in the Big Bang. Unlike $^{7}$Li, the less abundant $^{6}$Li has long been recognized as a nucleosynthetic "orphan".

- ❑ $^{6}$Li is made neither in the Big Bang nor in stars
- ❑ The stellar thermonuclear processes destroy $^{6}$Li

$^{6}$Li, $^{9}$Be and $^{10}$B are pure spallative products. Be is very precious to astrophysics since it is one stable isotope. Among LiBeB, Li and B have two stable isotopes ($^{7}$Li and $^{10}$B).

# Background

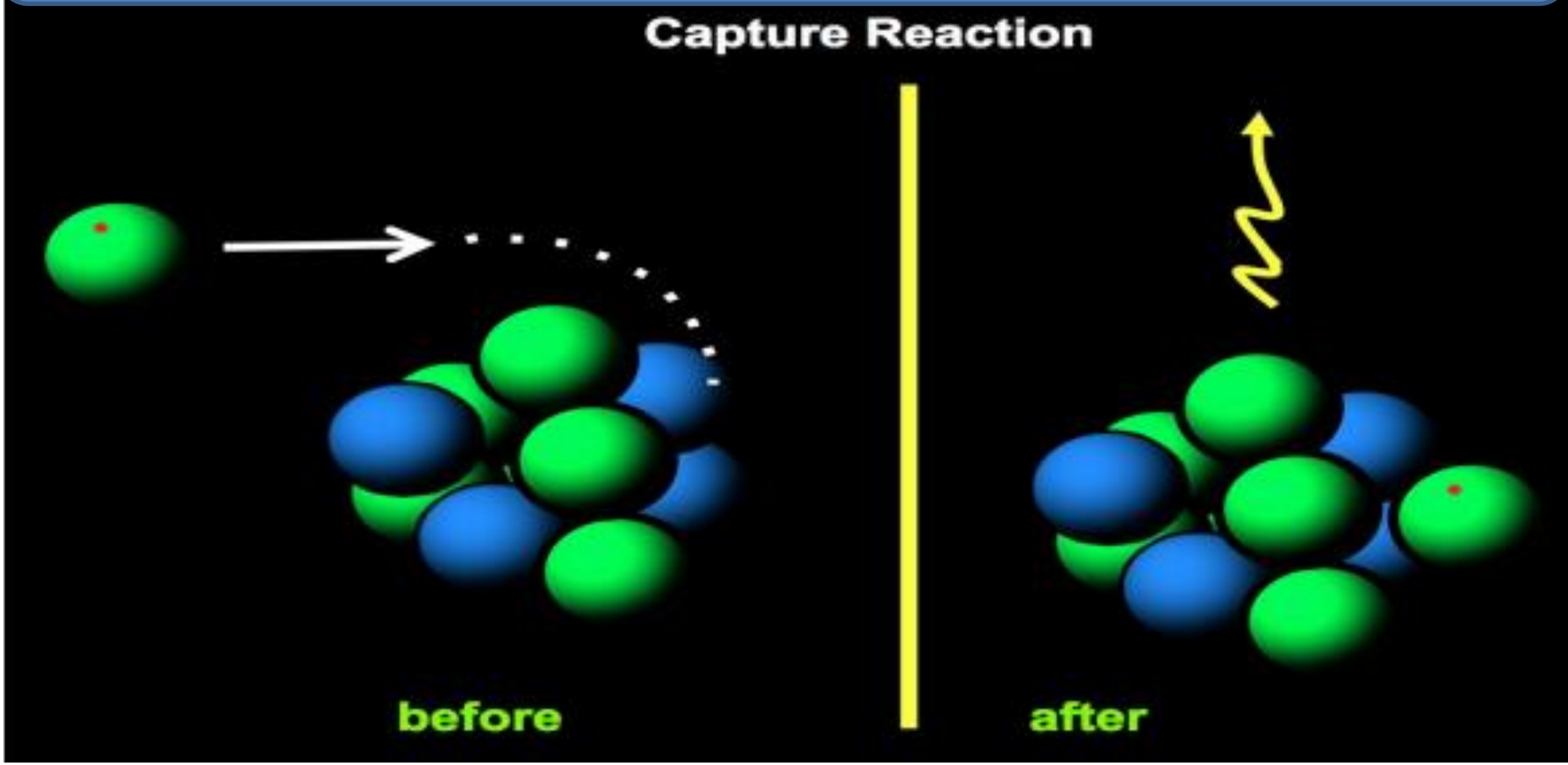
**Nuclear Spallation: The high-energy nuclear reaction in which a target nucleus struck by an incident particle of energy greater than about 50 million electron volts (MeV) ejects numerous lighter particles and becomes a product nucleus correspondingly lighter than the original nucleus.**




https://link.springer.com/referenceworkentry/10.1007%2F978-3-642-13271-1_30

# Objective of present study

**Radiative Capture Reaction: The reaction in which an atomic nucleus fuses with one or more nucleons or nuclei with the emission of electromagnetic radiation.**




https://www.phy.ornl.gov/groups/astro/measurements/capture.html

# Motivation

**Among the LiBeB we have considered the formation of $^{10}B$ by the radiative capture process.**

- $^{10}B$ is formed either by the spallation process or by the radiative capture process
- The radiative capture process is responsible for the low abundance of $^{9}Be$
- The $^{9}Be$ $(p, \gamma)$ $^{10}B$ reaction might act as an intermediate pathway between the p-p chain and the CNO cycle . It also plays a role in stellar nucleosynthesis of light elements in the *p* shell
- We applied the potential model (PM) to study the $^{9}Be$ $(p, \gamma)$ $^{10}B$ reaction, the nuclear structure of $^{10}B$ and the role of the associated reaction in LiBeB abundance determination
- The PM assumes that the interacting nuclei are two structure-less particles, which are interacting via the potential

# Motivation

- The solution obtained by employing the PM is often simpler and it is good enough for the calculation of nuclear cross-section
- For the first time, we employed the modified form of Woods-Saxon potential for the calculation of bound state wave functions of $^{10}B$ while for the scattering state, we used the conventional Woods-Saxon potential
- We have considered the transitions from resonance states $^{\pi}J = {}^{-}2$ and $^{\pi}J = {}^{-}1$ to the ground and first three excited states, respectively, in the $^{10}B$
- In contrast to other models, the computed results, based on the modified form of Woods-Saxon potential, show a better comparison with the experimental data

# Formalism

## Model for Calculations

We used the potential model for the calculation of proton capture cross-section and associated nuclear rates of $^{9}$Be ($p$, $\gamma$) $^{10}$B reaction. PM reduces many-particle system into two structure-less-particle system. For calculation of proton capture cross-section by PM, the total potential of the colliding particles is considered as

$$V(r) = V_N(r) + V_C(r), \qquad (1)$$

$$V_N(r) = \left[V_0 + V_{LS}(L \cdot S)\frac{1}{m_\pi^2 r}\frac{d}{dr}\right]\frac{1}{1 + q \cdot (\exp\frac{r-R_N}{a})}, \qquad (2)$$

Eq. (2) is the modified form of Woods-Saxon potential. We introduce the $q$ (the modification parameter in Wood-Saxon potential) for the first time to calculate the bound state wave function for the radiative capture processes. It is to be noted that for higher value of $q$ the Woods-Saxon potential approaches the Gaussian potential.

# Formalism

## Model for Calculations

$$V_C(r) = \begin{cases} \frac{\hbar c}{2} \frac{Z_1 Z_2 \alpha}{R_c}\left(3 - \frac{r^2}{R_c^2}\right) & \text{if } r \leq R_c, \\ \hbar c \frac{Z_1 Z_2 \alpha}{r} & \text{if } r \geq R_c, \end{cases} \quad (3)$$

Eq. (3) is the Coulomb part of the total potential $V(r)$

$$\frac{d^2}{dr^2}\varphi_L(r) + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\hbar^2 L(L+1)}{2\mu r^2}\right]\varphi_L(r) = 0, \quad (4)$$

Eq. (4) is the radial parts of Schrodinger equation including the bound state energy $E$, total potential $V(r)$. The asymptotic behavior of the bound state wave function is defined as

$$\varphi_L(r) \xrightarrow{r\to\infty} C_w W_{-\eta_0, L+\frac{1}{2}}(2\kappa_0 r), \quad (5)$$

where $\boldsymbol{C}_w$ is the Asymptotic normalization constant and $W_{-\eta_0, L+\frac{1}{2}}(2k_0 r)$ is the Whittaker function.

$$\varphi_L(r) \xrightarrow{r\to\infty} \cos\delta_L F_L(kr) + \sin\delta_L G_L(kr), \quad (6)$$

Eq. (6) is the scattering wave function with scattering phase shift $(\delta_L)$. $F_L(kr)$ and $G_L(kr)$ are the regular and irregular function.

# Formalism

## Astrophysical S-factor *S*(*E*)

The astrophysical S-factor *S*(*E*) is a rescaling of a nuclear reaction cross-section *σ*(*E*) to account for the Coulomb repulsion between the charged reactants. S-factor determines the rates of nuclear fusion reactions that occur in the core of stars.

$$S(E) = \sigma(E)E\exp(2\pi\eta), \qquad (7)$$

In Eq. (7) *E* is the center of mass energy for the interacting particles and *η* is Sommerfeld parameter

$$\eta = Z_1 Z_2 e^2/\hbar v, \qquad (8)$$

The cross-section is sum over the total angular momentum $J_f$ and electric multipolarity λ

$$\sigma(E) = \sum_{J_f,\lambda} \sigma_{\lambda,J_f}(E), \qquad (9)$$

# Formalism

## Model for Calculations

The summation term of Eq. (9) is defined as

$$\sigma_{\lambda,J_f}(E) = 8\pi\alpha\frac{c}{\upsilon k^2}\Big[Z_1\Big(\frac{A_2}{A}\Big)^\lambda + Z_2\Big(-\frac{A_1}{A}\Big)^\lambda\Big]^2 C^2 S_{J_f}\times$$
$$\sum_{J_i,I,l_i}\frac{(\kappa_\gamma)^{2\lambda+1}}{[(2\lambda+1)!!]^2}\frac{(\lambda+1)(2\lambda+1)}{\lambda}\times$$
$$\frac{(2l_i+1)(2l_f+1)(2J_f+1)}{(2I_1+2)(2I_2+1)}\begin{pmatrix} l_f & \lambda & l_i \\ 0 & 0 & 0\end{pmatrix}^2\times$$
$$\begin{Bmatrix} J_i & l_i & I \\ l_f & J_f & \lambda\end{Bmatrix}^2(2J_i+1)\Big(\int_0^\infty \varphi_i(r)r^\lambda\varphi_f(r)dr\Big)^2. \tag{10}$$

The $C^2S_{Jf}$ is called the spectroscopic factor (SF), which plays an important role in nuclear astrophysics. It is the ratio of measured cross-section to the computed cross-section. $\lambda$ ($J_f$ -$J_i$ ) is the multipolarity. It is the angular momentum that is carried by the emitted photon.

# Formalism

## Model for Calculations

The nuclear rates for the $p + {}^{9}\mathrm{Be} \longrightarrow {}^{10}\mathrm{B} + \gamma$ process is defined below,

$$N_A\langle\sigma v\rangle = N_A\left(\frac{8}{\pi\mu(k_BT)^3}\right)^{1/2}\times \int_0^{E_0}\sigma(E)E\exp(-E/k_BT)dE. \quad (11)$$

- The major step for the calculation of the astrophysical S-factor is to analyze the elastic scattering phase shift of a given data or similar data as a function of energy
- The potential is introduced for the analysis of phase shift and binding energies
- The total cross-section of the radiative capture process is calculated as a function of energy for the selected potential
- Finally, the astrophysical S-factor *S*(*E*) and nuclear rate of the thermonuclear reaction are calculated

# Results and Discussion

## Electric Dipole Transitions

- The emitted electromagnetic radiation can be classified according to the change in angular momentum, if the angular momentum changes by 1, 2, … (dipole transition, quadrupole transition,…)
- Further dipole transition may either be electric (parity is not conserved) or magnetic (parity conserved)
- Among them the electric dipole transitions (*E1*) are more intense than magnetic dipole transitions (*M1*). From these assumptions one obtains the Weisskopf estimates for the $\gamma$-ray transition probabilities, which we mention below*

$$\lambda_W(\mathrm{E1})\hbar = 6.8 \times 10^{-2} A^{2/3} E_\gamma^3, \qquad \lambda_W(\mathrm{M1})\hbar = 2.1 \times 10^{-2} E_\gamma^3$$
$$\lambda_W(\mathrm{E2})\hbar = 4.9 \times 10^{-8} A^{4/3} E_\gamma^5, \qquad \lambda_W(\mathrm{M2})\hbar = 1.5 \times 10^{-8} A^{2/3} E_\gamma^5$$
$$\lambda_W(\mathrm{E3})\hbar = 2.3 \times 10^{-14} A^2 E_\gamma^7, \qquad \lambda_W(\mathrm{M3})\hbar = 6.8 \times 10^{-15} A^{4/3} E_\gamma^7$$

***C. Iliadis, Nuclear physics of stars. John Wiley & Sons, 2015.***

# Results and Discussion

## Spectroscopic Study of $^{9}Be\ (p, \gamma)^{10}B$

- The aim of our present studies is to know about the spectroscopic properties of $^{10}B$ using the PM approach
- For the calculation of astrophysical S-factor we used Eq. (7)
- We considered only the $E1$ transitions from the initial state to low-lying states of the $^{10}B$
- There are three low-lying bound states in $^{10}B$ where $E1$ resonance transitions (depends on the nuclear potential) are possible from the scattering state $^{\pi}J = {}^{-}1$. We further considered the $E1$ direct transition (described by the regular Coulomb function) with zero phase shift from the scattering $^{\pi}J = {}^{-}2$ to the ground state of $^{10}B$
- The potential parameters were selected for V(r) to regenerate the experimental data of the astrophysical S-factor. The parameters are mentioned in Table 2
- Finally, we used the computed data of the S-factor and calculate the nuclear rates ( Eq. 11)

# Results and Discussion

## Spectroscopic Study of $^{9}Be(p, \gamma)^{10}B$

Table. 1: The low-lying bound states of $^{10}B$ *

| Bound states | Binding energy | Proton orbital |
|---|---|---|
| $^{\pi}J_{f}$ | $\mathcal{E}_{f}$ (MeV) | $L_{Jf}$ |
| $^{+}3$ | 6.58 | $P_{3/2}$ |
| $^{+}1$ | 5.86 | $P_{3/2}$ |
| $^{+}0$ | 4.84 | $P_{3/2}$ |
| $^{+}1$ | 4.42 | $P_{3/2}$ |

* https://nucldata.tunl.duke.edu/

# Results and Discussion

## Parameters for the calculation of nuclear cross-section and nuclear rates

Table. 2: The parameters of potentials Eq. (1). The first column represents the states while the subsequent columns represent the binding energy, depth of central potential, spin-orbit potential, diffuseness parameter, the nuclear and Coulomb radii, respectively. The last column gives the modification parameter in Woods-Saxon potential.

| State | $\varepsilon_f$ MeV * | $V_0$ MeV | $V_{LS}$ MeV | a fm | $R_N$ fm | $R_C$ fm | q |
|---|---|---|---|---|---|---|---|
| Bound $^{+}1$ | 5.86 | 112.66 | 12.4 | 0.55 | 2.371 | 2.371 | 05.0 |
| $^{+}1$ | 4.84 | 170.50 | 12.4 | 0.50 | 2.413 | 2.413 | 08.0 |
| $^{+}1$ | 4.23 | 234.66 | 12.4 | 0.50 | 2.336 | 2.336 | 10.0 |
| Resonance $^{-}1$ | | 63.5 0 | | 65 | 2.600 | 2.600 | 1 |

* https://nucldata.tunl.duke.edu/

# Results and Discussion

## Parameters for the calculation of nuclear cross-section and nuclear rates

Table. 3: The parameters of potentials Eq. (1) for the direct transitions. The first column represents the states while the subsequent columns represent the binding energy, depth of central potential, spin-orbit potential, diffuseness parameter, the nuclear and Coulomb radii, respectively. The last column gives the modification parameter in Woods-Saxon potential.

| State | $\varepsilon_f$ MeV* | $V_0$ MeV | $V_{LS}$ MeV | a fm | $R_N$ fm | $R_C$ fm | q |
|---|---|---|---|---|---|---|---|
| Bound $^{+}3$ | 6.5867 | 60.68 | 12.4 | 0.55 | 2.69 | 2.69 | 1.2 |
| Resonance $^{-}2$ | | 70.20 | | 0.50 | 2.58 | 2.58 | 1 |

* https://nucldata.tunl.duke.edu/

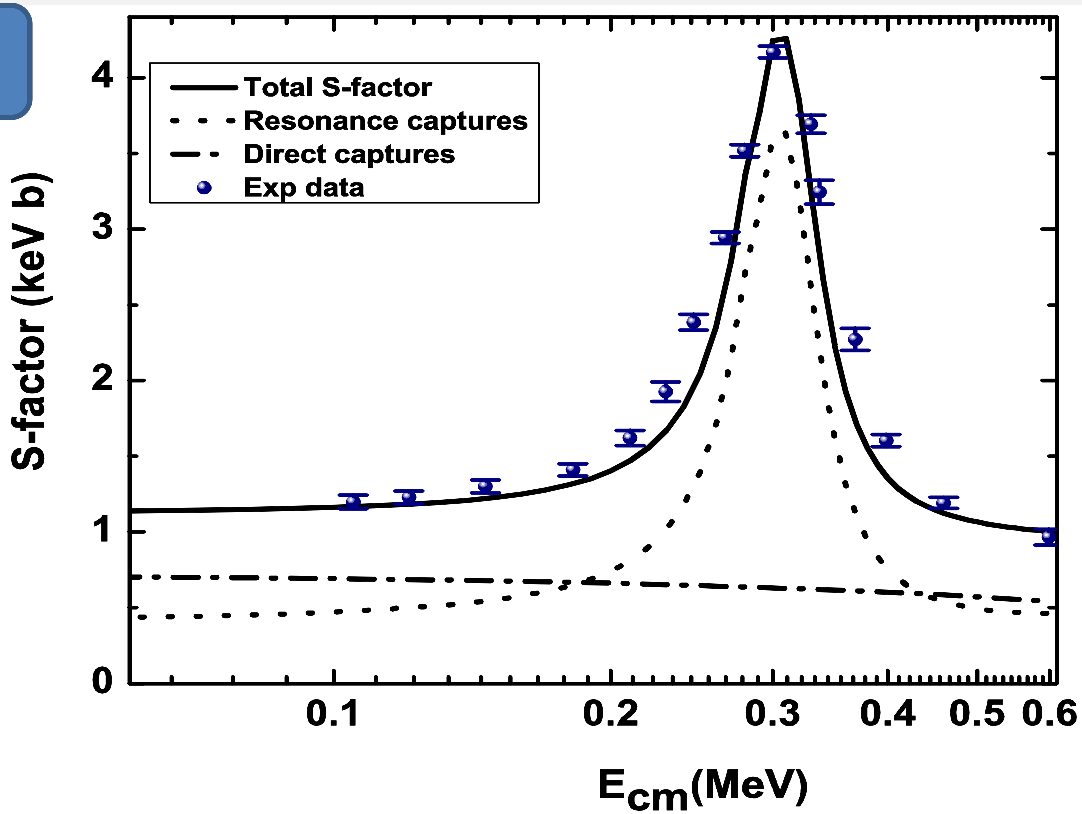
## Possible *E1* Transitions

Table. 4: Possible *E1* transitions to the ground and bound states from the resonance states

| Resonance States | | | | Bound states | | |
|---|---|---|---|---|---|---|
| $^{\pi}J_i$ | $E_r$ MeV | Width keV | $l_i$ | $^{\pi}J_f$ | $\varepsilon_f$ MeV | $l_f$ |
| $^{-}1$ | 0.30 | 120 | *s* | $^{+}1$ | 5.86 | *P* |
| | | | | $^{+}0$ | 4.84 | *P* |
| | | | | $^{+}1$ | 4.23 | *P* |
| $^{-}2$ | 0.90 | 210 | *S* | $^{+}3$ | 6.58 | *p* |

* https://nucldata.tunl.duke.edu/

## Astrophysical S-factor




The solid line shows our calculated S-factor (the sum of direct and resonance transitions), the filled circles show the experimental results, the dotted line shows the resonance transitions from $^{\pi}J = {}^{-}1$ to first three excited states of $^{10}B$ and the dash-dotted line show the direct capture transitions from $^{\pi}J = {}^{-}2$ to the ground state.

**D. Zahnow, et al., Nucl. Phys. A 589 (1995) 95**

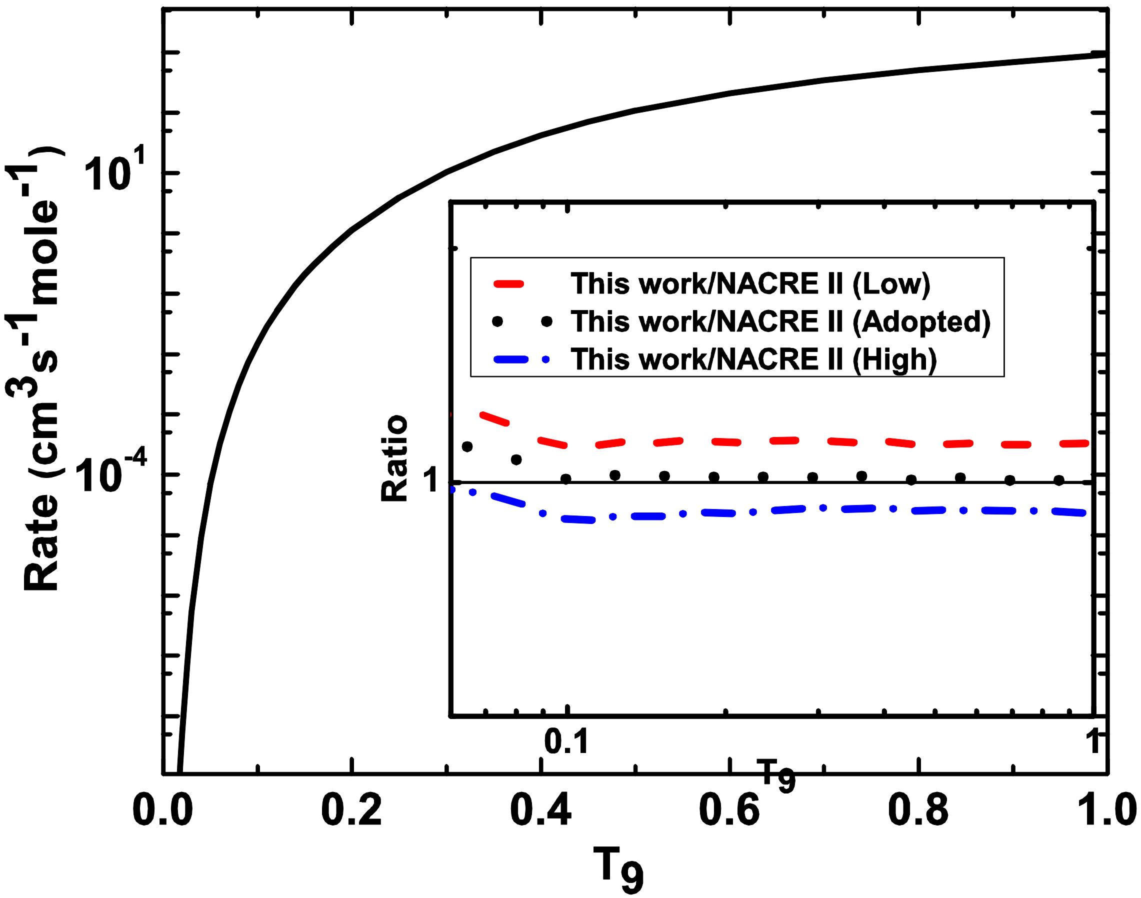
# Results and Discussion

## The Spectroscopic Factor (SF)

- The $C^2S_{J_f}$ shows the extent up to which a theoretical model behaves like a single particle
- The present value of SFs for the transitions $^{\pi}J = {}^{-}1$ to the first, second and third excited state of $^{10}B$ are 0.44, 0.36 and 0.11, respectively
- In contrast the reported data of NACRE II* obtained the SFs 0.018, 0.017 and 0.018, respectively
- Therefore, the present calculations for cross-section/astrophysical S-factor of $^9$Be ($p$, $\gamma$) $^{10}$B, based on the modified form of Woods-Saxon potential, are in better compassion with the experimental results

*Y. Xu, et al., Nucl. Phys. A 918 (2013) 61

## Capture rates




The solid line show our calculated rates. The dashed (red), dotted (black), and dash-dotted (blue) lines show the ratio of our calculated rates to the low, adopted and high rates of NACRE II*, respectively.

*Y. Xu, et al., Nucl. Phys. A 918 (2013) 61

# Conclusion

- In this work, we used the PM for the calculation of the astrophysical S-factor for both resonant and direct transitions.
- For the calculation of bound state wave functions, we modified the Woods-Saxon potential while for the scattering wave function we used the conventional form of Woods-Saxon potential where the modification parameter was equal to unity.
- The computed result for astrophysical S-factor shows a better agreement with the experimental data.
- Based on the astrophysical S-factor, we computed the nuclear rates for the $^{9}$Be ($p$, $\gamma$) $^{10}$B reaction which shows a better comparison with the adopted rates of NACRE II at high temperatures.

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# Thank You