Evolution of the Symmetry Energy and Skins of Mirror Nuclei

M.K. Gaidarov, A.N. Antonov, D.N. Kadrev

Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria

I. Moumene

High Energy Physics and Astrophysics Laboratory, Faculty of Science Semlalia, Cadi Ayyad University, P.O.B. 2390, Marrakesh, Morocco

P. Sarriguren

Instituto de Estructura de la Materia, IEM-CSIC, Serrano 123,

E-28006 Madrid, Spain

E. Moya de Guerra

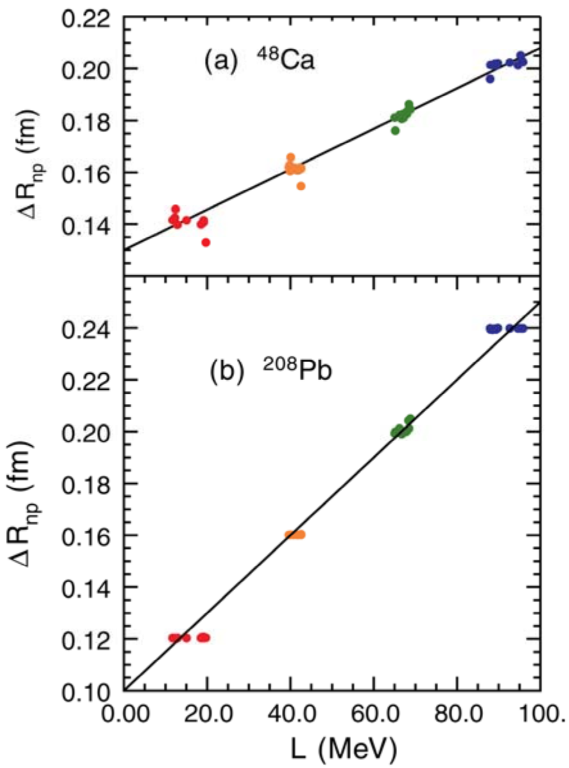
Departamento de Estructura de la Materia, Física Térmica y Electrónica, and IPARCOS, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Madrid E-28040, Spain

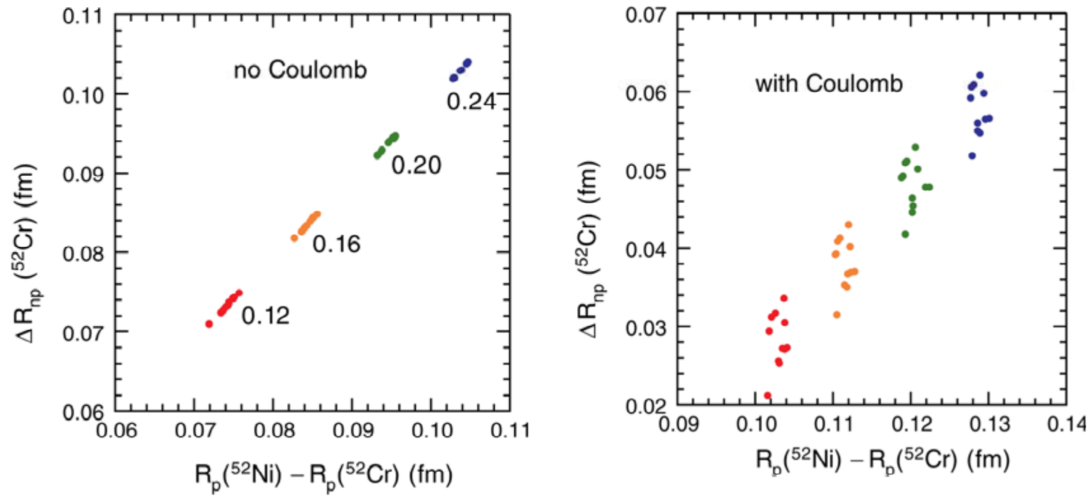
Motivation

B. Alex Brown, Phys. Rev. Lett. **119**, 122502 (2017)

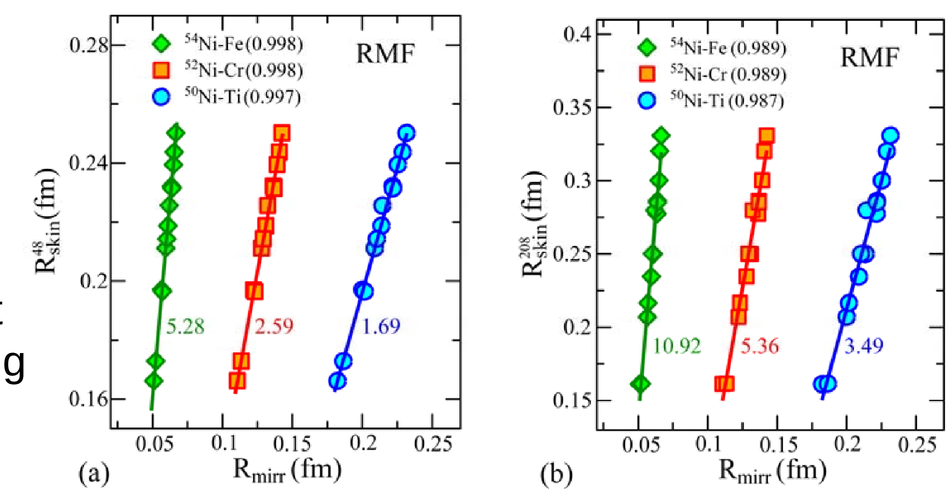
Junjie Yang and J. Piekarewicz, Phys. Rev. C **97**, 014314 (2018)

48 Skyrme functionals





14 relativistic energy density functionals



F. Sammarruca, Front. Phys. **6:90** (2018)

Microscopic EoS obtained from self-consistent Brueckner-Hartree-Fock calculations employing high-precision chiral few-nucleon forces

In the present work:

- We focus on nuclei in the mass region A=48-60, in which the Ni isotopes and respective mirror nuclei are studied.
- We search for possible correlations between the neutron-skin thickness and the EOS parameters (symmetry energy, pressure, asymmetric compressibility) for various Ni isotopes and their corresponding mirror partners.
- We calculate the proton skins of Argon isotopes (A=32-40) and predictions for them are made, in comparison with the empirical data and the microscopic results with high-precision chiral few-nucleon forces.
- We inspect the relation between the neutron skin and the difference of the proton radii of the corresponding mirror nuclei for the Z=20 and Z=28 isotopic chains and for N=14 and N=50 isotonic chains.
- We pay particular attention to the Z=20 isotopic chain, inspired by the new experiment on this nucleus (CREX) that is ongoing at JLab.

Calculations:

Densities and radii: within a self-consistent HFB method by using the cylindrical transformed deformed HO basis (HFBTHO)

Symmetry energy and related quantities: in the CDFM framework by use of Brueckner and Skyrme EDFs for nuclear matter with SLy4 and SkM* effective forces.

Mirror Nuclei

The neutron-and proton-skin thicknesses:

$$\Delta R_n = R_n(Z,N) - R_p(Z,N) \tag{1}$$

$$\Delta R_p = R_p(Z,N) - R_n(Z,N) \tag{2}$$

$$\Delta R_n = -\Delta R_p \tag{3}$$

Under the assumption of exact charge symmetry:

$$R_n(Z,N) = R_p(N,Z) \tag{4}$$

$$\Delta R_{mirr} = R_p(N,Z) - R_p(Z,N) \tag{5}$$

Thus, in the case of the exact charge symmetry:

$$\Delta R_n = R_p(N,Z) - R_p(Z,N) = \Delta R_{mirr} \tag{6}$$

The key EOS parameters in nuclear matter (NM)

$$E(\rho,\delta) = E(\rho,0) + S(\rho)\delta^2 + O(\delta^4) + \ldots$$

$$\delta = (N-Z)/A$$

$$E(\rho,0) = E_0 + \frac{K}{18\rho_0^2}(\rho-\rho_0)^2 + \ldots$$

$$S(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\delta=0} =$$

$$= a_4 + \frac{p_0}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K}{18\rho_0^2}(\rho-\rho_0)^2 + \ldots$$

a_4 is the symmetry energy at $\rho = \rho_0$

Weizsacker mass formula

$$E(N,Z) = E_{mac} + E_{mic} = E_V + E_S + E_a + E_C + E_{mic} =$$

$$= -a_V A + a_S A^{2/3} + a_a \frac{(N-Z)^2}{A} + a_C \frac{Z^2}{A^{1/3}} + E_{mic}$$

Symmetry pressure

$$p_0^{NM} = \rho_0^2 \frac{\partial S}{\partial\rho}\bigg|_{\rho=\rho_0}$$

Slope parameter

$$L^{NM} = \frac{3p_0^{NM}}{\rho_0}$$

Asymmetric compressibility

$$\Delta K^{NM} = 9\rho_0^2 \frac{\partial^2 S}{\partial\rho^2}\bigg|_{\rho=\rho_0}$$

Brueckner energy-density functional for infinite NM

$$V(x) = AV_0(x) + V_C - V_{CO}\ , \text{ where}$$

$$V_0(x) = 37.53\left[(1+\delta)^{5/3} + (1-\delta)^{5/3}\right]\rho_0^{2/3}(x) +$$

$$+ b_1\rho_0(x) + b_2\rho_0^{4/3}(x) + b_3\rho_0^{5/3}(x) + \delta^2\left[b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x)\right]$$

$$b_1 = -741.28\ ;\ b_2 = 1179.89\ ;\ b_3 = -467.54\ ;\ b_4 = 148.26\ ;\ b_5 = 372.84\ ;\ b_6 = -769.57$$

$$V_C = \frac{3}{5}\frac{Z^2e^2}{x} \qquad V_{CO} = 0.7386\ Ze^2(3Z/4\pi x^3)^{1/3}$$

$$S^{NM}(x) = 41.7\,\rho_0^{2/3}(x) + b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x)$$

$$p_0^{NM} = 27.8\,\rho_0^{5/3}(x) + b_4\rho_0^{2}(x) + \frac{4}{3}b_5\rho_0^{7/3}(x) + \frac{5}{3}b_6\rho_0^{8/3}(x)$$

$$\Delta K^{NM} = -83.4\,\rho_0^{2/3}(x) + 4\,b_5\rho_0^{4/3}(x) + 10\,b_6\rho_0^{5/3}(x)$$

- For Skyrme EDF:

$$
\begin{aligned}
\mathcal{E}(r,T) &= \frac{\hbar^2}{2m_{n,k}}\tau_n + \frac{\hbar^2}{2m_{p,k}}\tau_p \\
&+ \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(x_0+\frac{1}{2}\right)(\rho_n^2+\rho_p^2)\right] \\
&+ \frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{x_3}{2}\right)\rho^2 - \left(x_3+\frac{1}{2}\right)(\rho_n^2+\rho_p^2)\right] \\
&+ \frac{1}{16}\left[3t_1\left(1+\frac{1}{2}x_1\right) - t_2\left(1+\frac{1}{2}x_2\right)\right](\nabla\rho)^2 \\
&- \frac{1}{16}\left[3t_1\left(x_1+\frac{1}{2}\right) + t_2\left(x_2+\frac{1}{2}\right)\right] \\
&\times \left[(\nabla\rho_n)^2 + (\nabla\rho_p)^2\right] + \mathcal{E}_c(r)
\end{aligned} \tag{7}
$$

(for infinite homogeneous NM only the first three lines contribute)

- t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, and α are the Skyrme parameters
- We use SkM* and SLy4 Skyrme interactions

CDFM (Sofia group 1979 - till now)

$$\rho(\mathbf{r},\mathbf{r}') = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_x(\mathbf{r},\mathbf{r}') \tag{12}$$

$$\rho_x(\mathbf{r},\mathbf{r}') = 3\rho_0(x)\frac{j_1(k_F(x)|\mathbf{r}-\mathbf{r}'|)}{(k_F(x)|\mathbf{r}-\mathbf{r}'|)}\Theta\left(x - \frac{|\mathbf{r}+\mathbf{r}'|}{2}\right) \tag{13}$$

$$\rho_0(x) = \frac{3A}{4\pi x^3} \tag{14}$$

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\beta}{x} \qquad \beta = \left(\frac{9\pi A}{8}\right)^{1/3} \simeq 1.52A^{1/3} \tag{15}$$

$$\rho(\mathbf{r}) = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x)\Theta(x - |\mathbf{r}|) \tag{16}$$

$$\text{At } \frac{d\rho}{dr} \leq 0: \qquad |\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)} \left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{17}$$

Symmetry energy parameters of finite nuclei in CDFM

$$S = \int_0^\infty dx \left|F(x)\right|^2 S^{NM}(x)$$

$$p_0 = \int_0^\infty dx \left|F(x)\right|^2 p_0^{NM}(x) = \int_0^\infty dx \left|F(x)\right|^2 \left[\rho_0^2(x) \left.\frac{\partial S}{\partial \rho}\right|_{\rho = \rho_0(x)} \right]$$

$$\Delta K = \int_0^\infty dx \left|F(x)\right|^2 \Delta K^{NM}(x) = \int_0^\infty dx \left|F(x)\right|^2 \left[9 \rho_0^2(x) \left.\frac{\partial^2 S}{\partial \rho^2}\right|_{\rho = \rho_0(x)} \right]$$

Neutron (R_n), proton (R_p), matter (R_m), and charge (R_c) rms radii (in fm) calculated with SLy4 force for Z=10 and Z=18 isotopic chains. The proton skins ΔR_p (in fm) are also shown in comparison with the results obtained in Ref. [24].

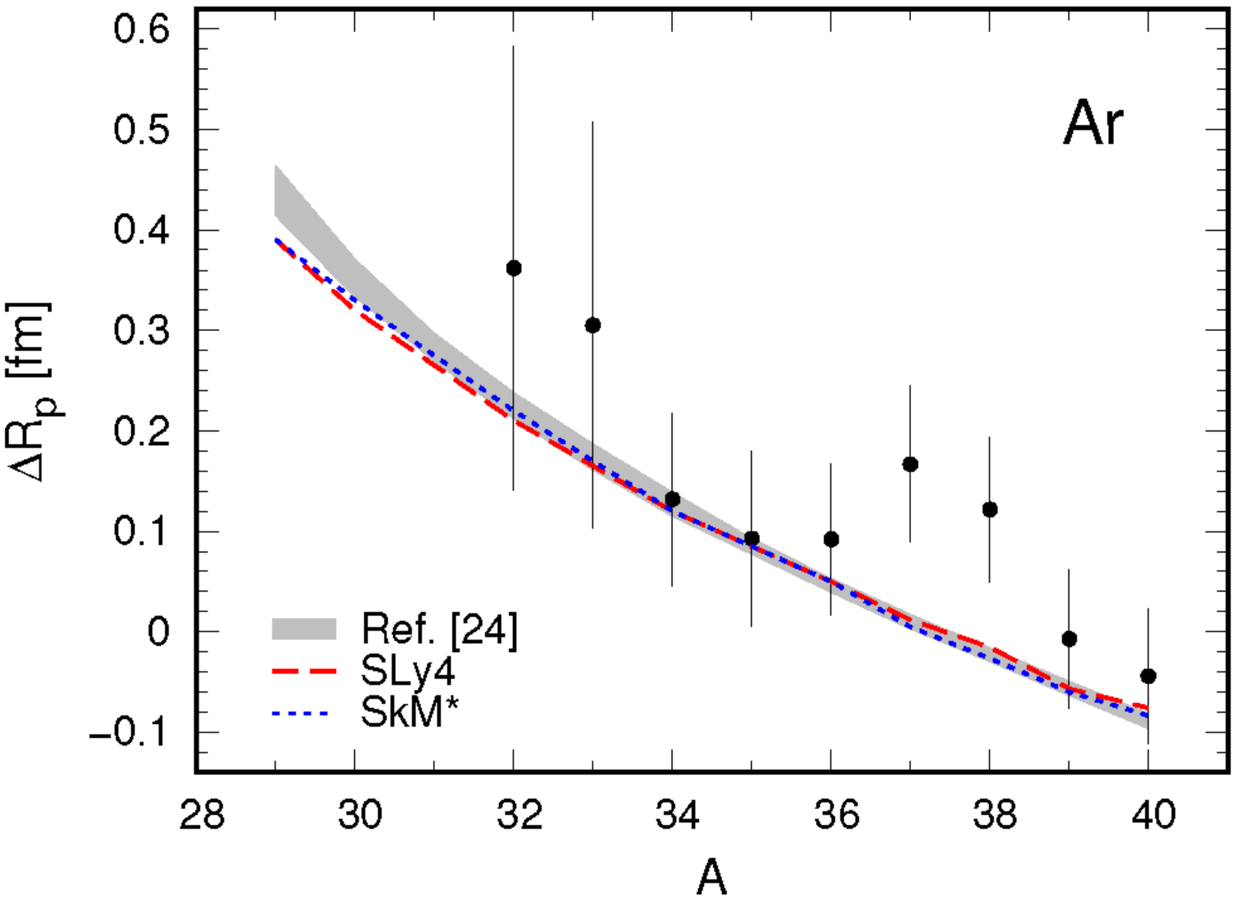
Nucleus	Z	N	R_n	R_p	R_m	R_c	ΔR_p	ΔR_p [24]
^{16}Ne	10	6	2.51	2.89	2.76	3.00	0.378	0.422 ± 0.022
^{18}Ne		8	2.67	2.85	2.77	2.96	0.175	0.186 ± 0.012
^{20}Ne		10	2.81	2.84	2.82	2.95	0.029	0.032 ± 0.006
^{30}Ar	18	12	3.00	3.32	3.20	3.42	0.323	0.352 ± 0.019
^{32}Ar		14	3.07	3.28	3.19	3.38	0.216	0.225 ± 0.013
^{34}Ar		16	3.17	3.29	3.23	3.39	0.123	0.127 ± 0.012
^{36}Ar		18	3.26	3.31	3.29	3.40	0.046	0.046 ± 0.007

[24] **F. Sammarruca**, Front. Phys. **6:90** (2018)

Predicted proton skins ΔR_p (in fm) for the same Z=10 and Z=18 nuclei (columns 1 and 2), neutron skins ΔR_n (in fm) of the corresponding mirror nuclei (columns 3 and 4), and ΔR_{mirr} (in fm) (column 5) calculated with SkM* force.

Nucleus	ΔR_p	Mirror nucleus	ΔR_n	ΔR_{mirr}
^{16}Ne	0.366	^{16}C	0.308	-0.360
^{18}Ne	0.160	^{18}O	0.111	-0.147
^{20}Ne	0.024	^{20}Ne	-0.024	0.000
^{30}Ar	0.327	^{30}Mg	0.238	-0.308
^{32}Ar	0.222	^{32}Si	0.136	-0.188
^{34}Ar	0.128	^{34}S	0.041	-0.088
^{36}Ar	0.044	^{36}Ar	-0.044	0.000

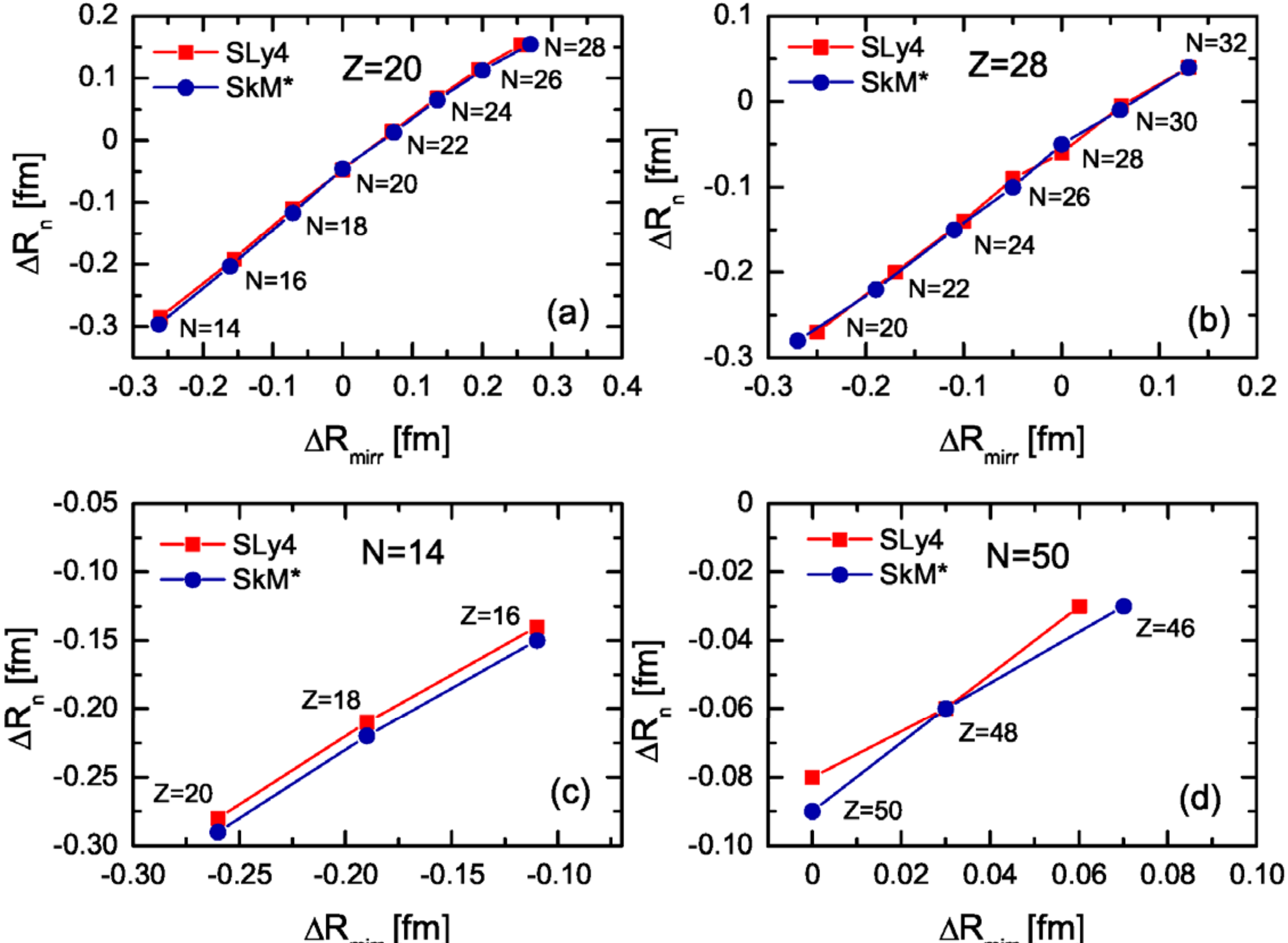
Predictions for Proton Skins



Ref. [24]: The EoS is based upon N^{3}LO two-nucleon forces (2NF) plus the leading 3NF

Experiment: **A. Ozawa *et al.***, Nucl. Phys. A **709**, 60 (2002)

Relation between the neutron skin ΔR_n and ΔR_{mirr} for the Z=20 (a) and Z=28 (b) isotopic chains and for the N=14 (c) and N=50 (d) isotonic chains



Linear fit of the curves:

$$\Delta R_n = c(\Delta R_{mirr}) + d \tag{1}$$

In the case of SkM* force:

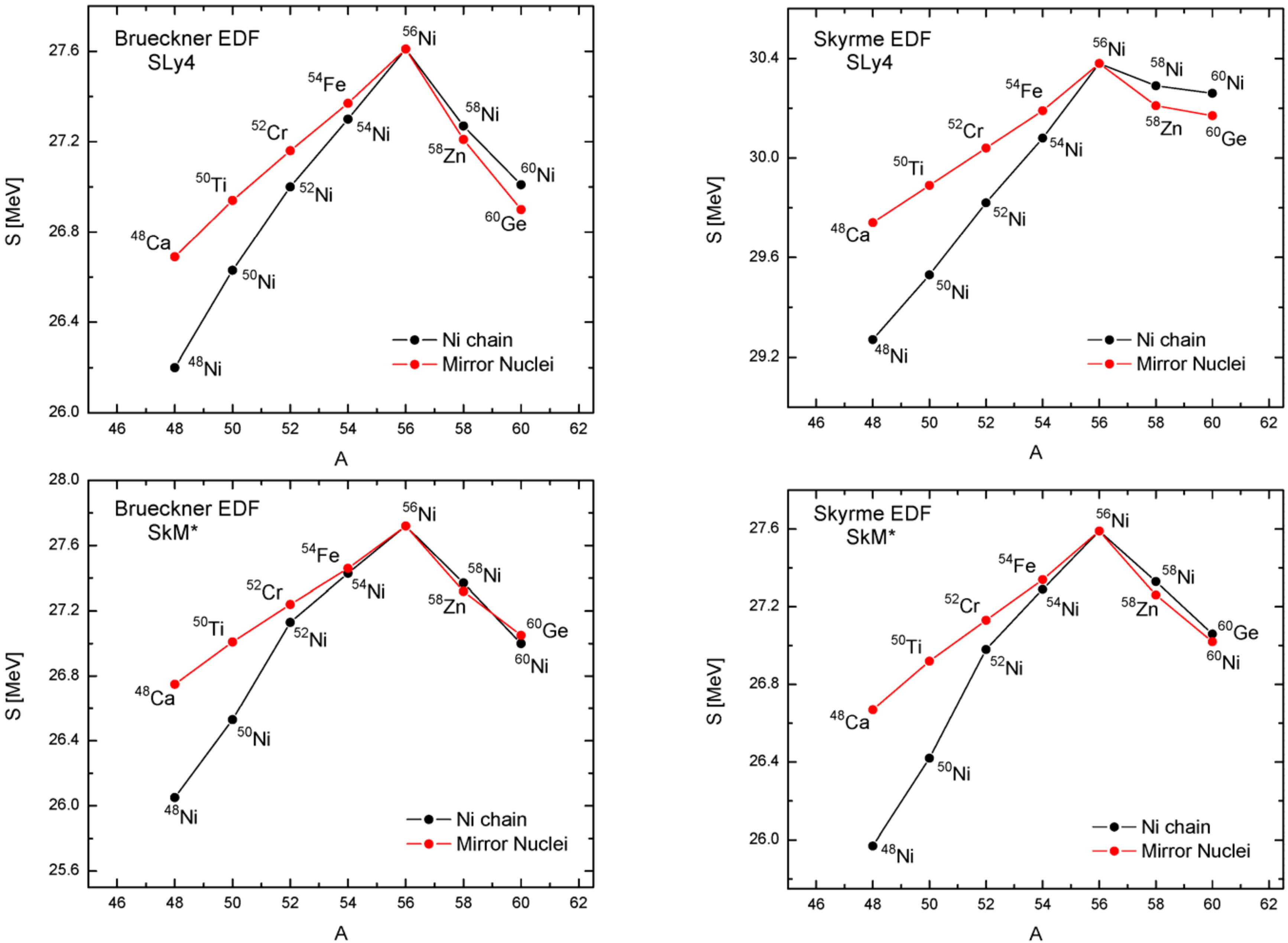
$$c = 0.866 \pm 0.037, \qquad d = -0.0633 \pm 0.0041 \tag{2}$$

In the case of SLy4 force:

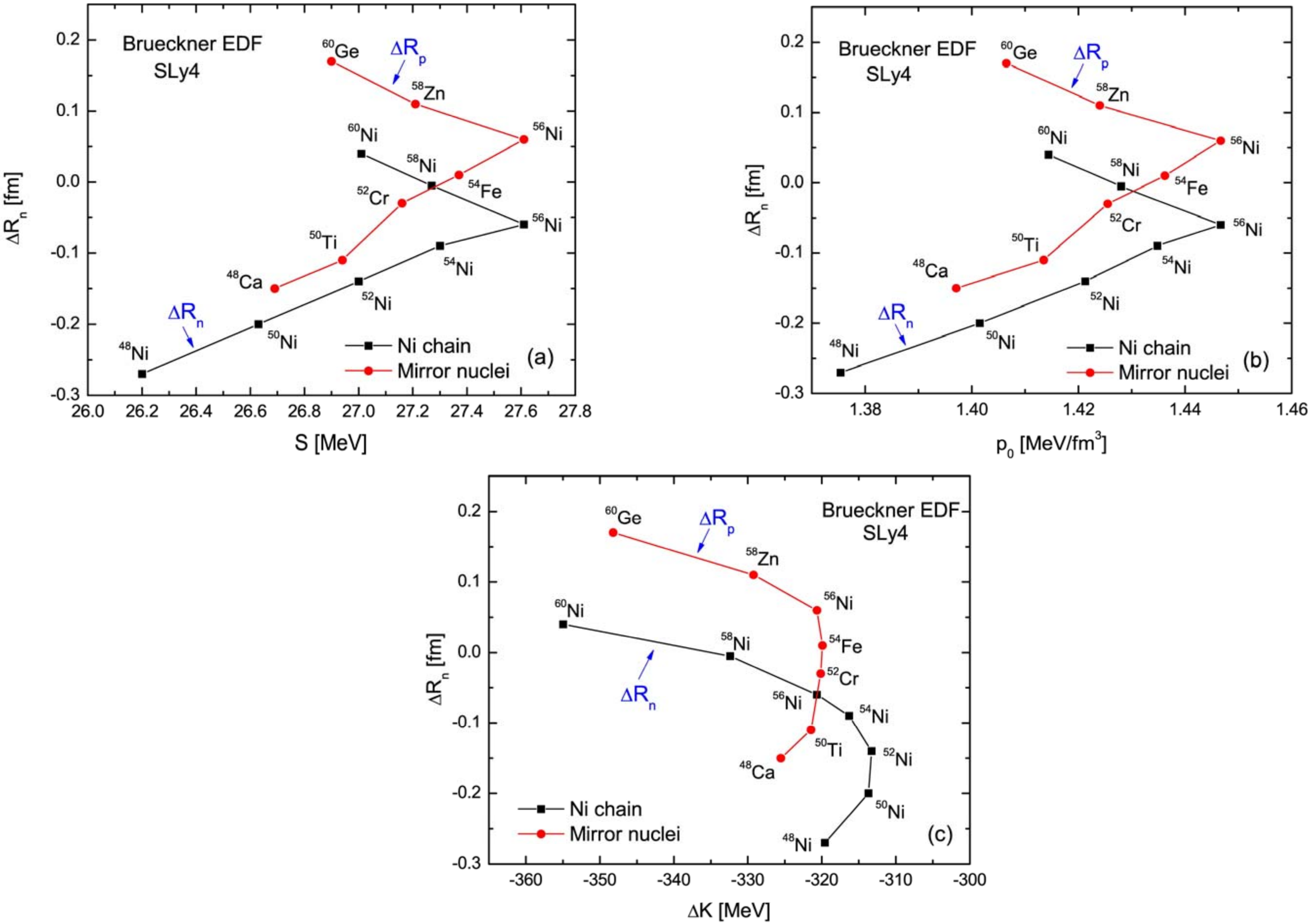
$$c = 0.862 \pm 0.042, \qquad d = -0.0575 \pm 0.0041 \tag{3}$$

In the limit of $\Delta R_{mirr} = R_p(N,Z) - R_p(Z,N) \rightarrow 0$ a small negative value of ΔR_n appears (Coulomb effects !)

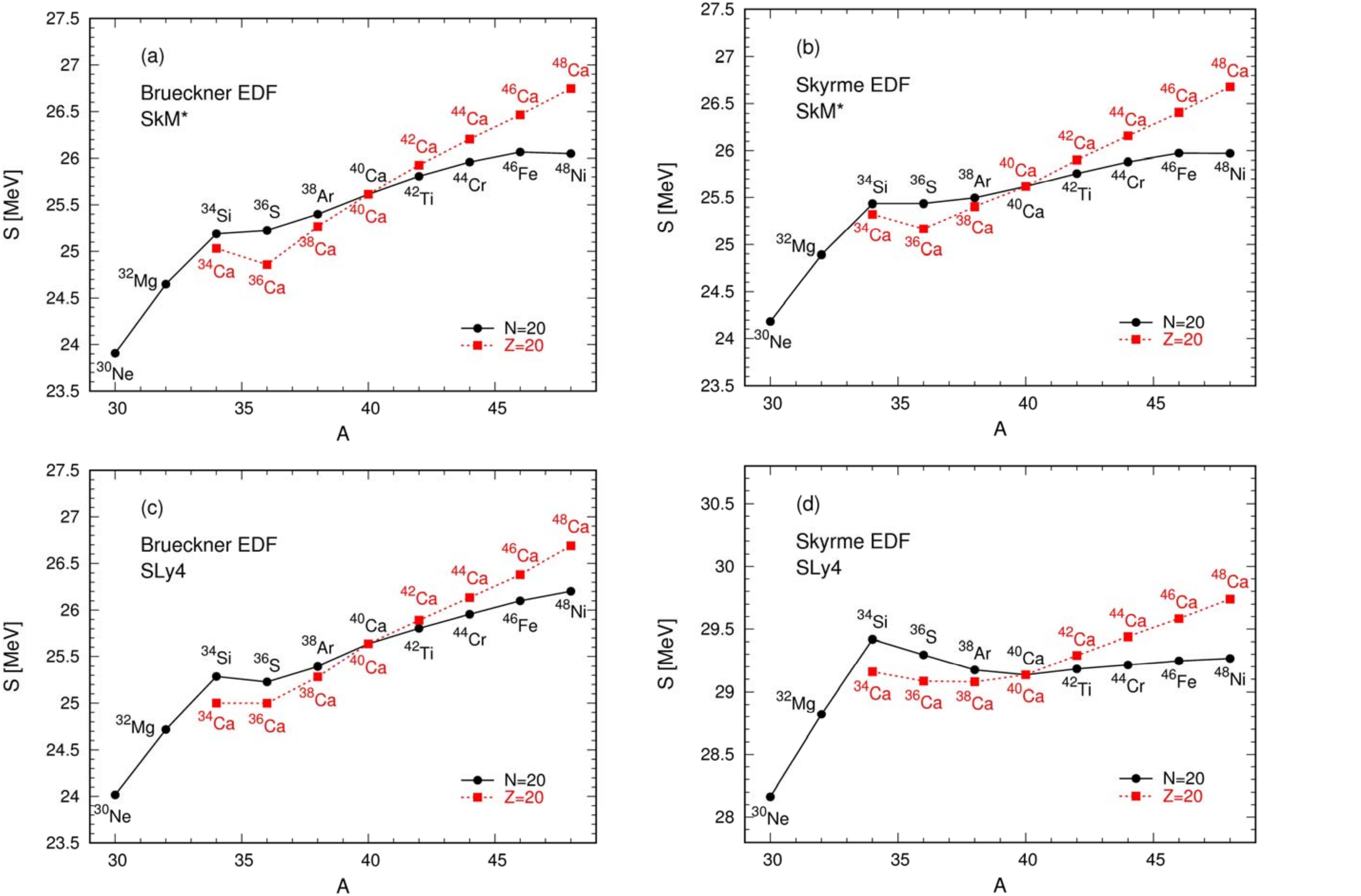
S versus *A* for the Ni isotopic chain (*Z*=28) with *A*=48-60 and the corresponding mirror nuclei



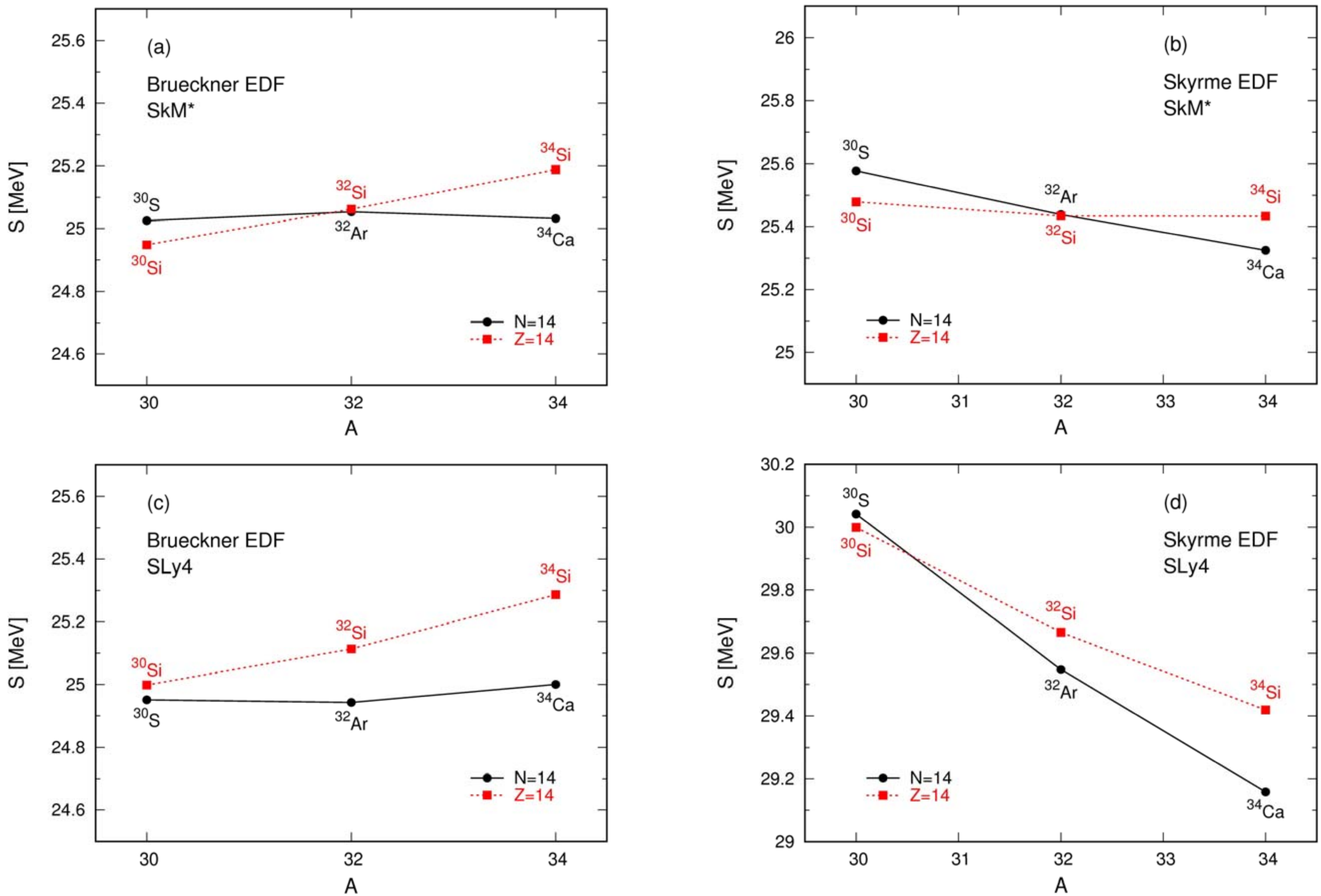
ΔR_n for Ni isotopes and $\Delta R_p = -\Delta R_n$ for their mirror nuclei as a function of the symmetry energy S, pressure p_0, and asymmetric compressibility ΔK



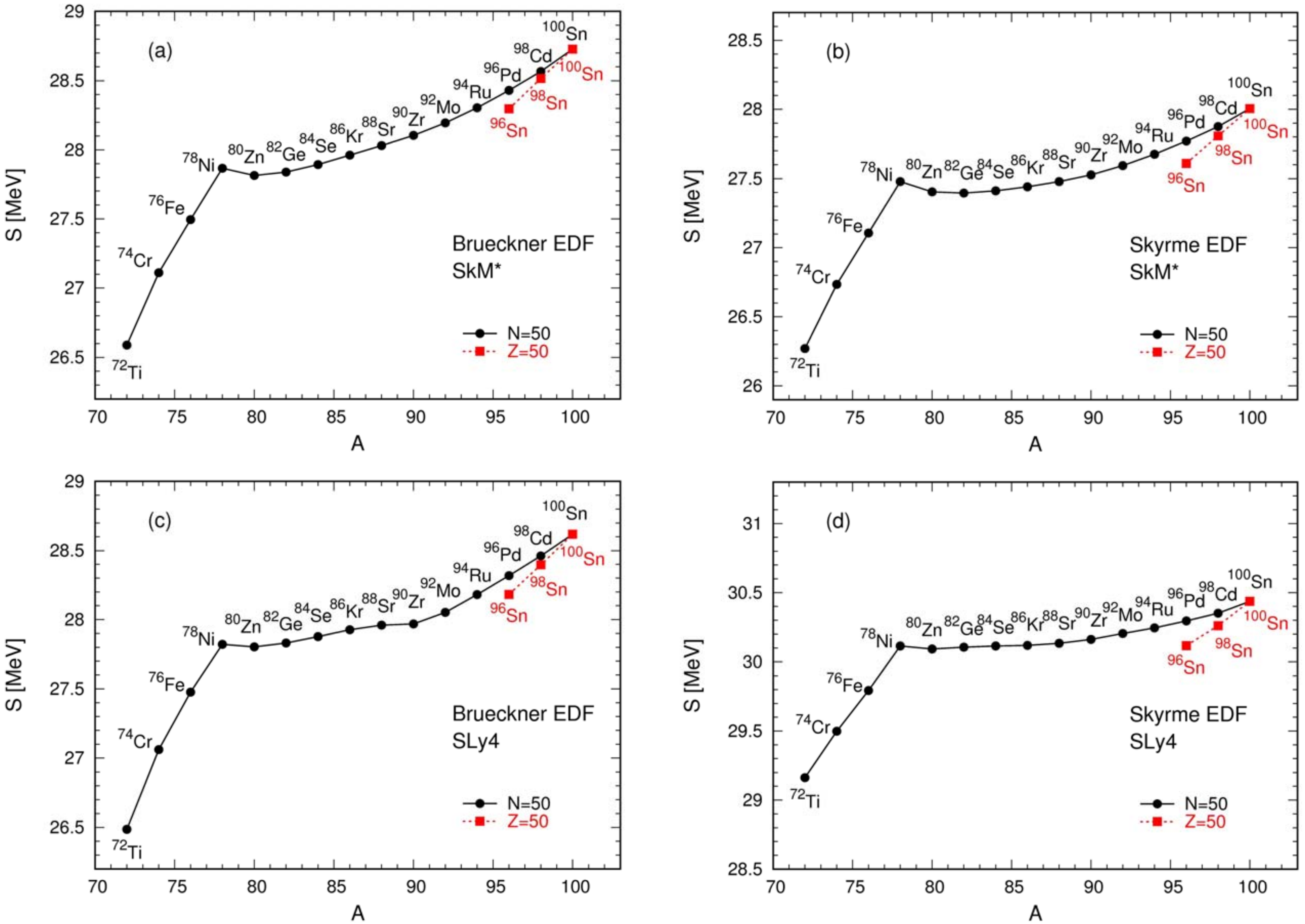
S versus *A* for Ca isotopes and their mirror nuclei



S versus *A* for *N*=14 isotones and their mirror Si isotopes



S versus *A* for *N*=50 isotones and their heaviest three mirror Sn isotopes



Conclusions

- The HFB method by using the cylindrical transformed deformed harmonic-oscillator basis has been applied to calculations of radii and skins for several mirror pairs in the middle mass range.

- The parameters of the EoS (S, p_0, ΔK) have been calculated for Ni isotopic chain with mass number A=48−60, as well as for nuclei with Z=20, N=14, and N=50 and their respective mirror nuclei using Brueckner and Skyrme EDFs for isospin asymmetric nuclear matter with two Skyrme-type forces, SkM* and SLy4, and the CDFM that links the properties of nuclear matter with the microscopic description of finite nuclei.

1. Due to Coulomb effects, the predicted proton skins are found larger than the neutron skins of the corresponding mirror nuclei. They compare reasonably well with available empirical data, for instance for Ar isotopes, that are also well described by chiral EFT-based EoS.

2. The studied relation between the neutron skin ΔR_n and the difference between the proton radii ΔR_{mirr} for a family of mirror pairs in the presence of Coulomb effects shows clearly a linear dependence. Thus, this appears to be an alternative way to explore neutron skins that may challenge experimentalists to perform high-precision measurements of charge radii of unstable neutron-rich isotopes.

3. We found in the case of Ni isotopic chain and the respective mirror nuclei a strong correlation between the neutron (proton)-skin thickness and the *S* and p_0 parameters of the EoS with a “kink” at double-magic ^{56}Ni, while the correlation with between ΔR_n (ΔR_p) and ΔK is less pronounced.

4. The evolution of the symmetry energy *S* with the mass number *A* of nuclei from *Z*=20, *Z*=28 and *N*=50 chains and their mirror nuclei exhibits similar behavior. The curves cross in each chain at the corresponding *N*=*Z* nucleus (^{40}Ca, ^{56}Ni, ^{100}Sn) and start to deviate from each other with the increase of the level of asymmetry |*N*-*Z*|.

M.K. Gaidarov, I. Moumene, A.N. Antonov, D.N. Kadrev, P. Sarriguren, E. Moya de Guerra, **Proton and Neutron Skins and Symmetry Energy of Mirror Nuclei**, Nucl. Phys. A 1004, 122061 (2020)