

Vortical excitations in nuclei: recent progress

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NSP2021, 2-4 June 2021, Konya-Turkey

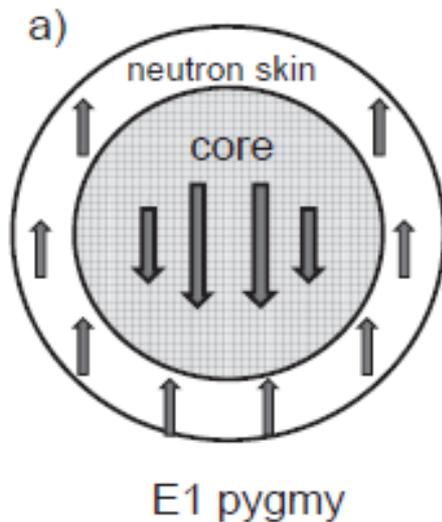
Content:

- ★ Exotic E1 excitations. Nuclear vorticity. J. Kvasil et al, PRC [84](#), 034303 (2011)
- ★ Toroidal dipole resonance (TDR) as a remarkable case of a vortical flow,
 - modern theoretical and experimental status V.O. Nesterenko et al, Phys. Atom. Nucl. [79](#), 842 (2016)
 - TDR vs PDR A. Repko et al PRC, [87](#), 024305 (2013), EPJA, [55](#), 242 (2019)
- ★ Individual toroidal states in light nuclei V.O. Nesterenko et al, PRL [120](#), 182501 (2018)
Y. Kanada-En'yo et al, PRC [95](#), 064319 (2017)
- ★ Search of vortical states in (e,e') V.O. Nesterenko et al, PRC [100](#), 064302 (2019).
- ★ Conclusions and outlook

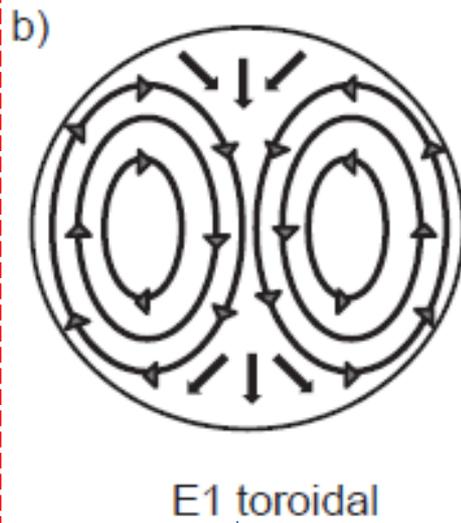
Exotic E1 excitations
Intrinsic nuclear vorticity

Exotic dipole resonances

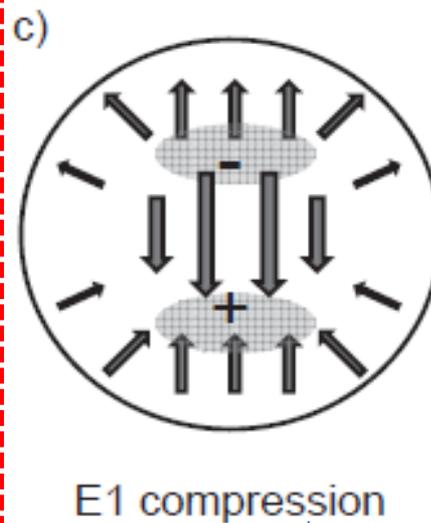
R. Mohan et al (1971),



V.M. Dubovik (1975)
S.F. Semenko (1981)



M.N. Harakeh (1977)
S. Stringari (1982)



Alternative source of information on nuclear incompressibility

Dominate in E1(T=0) excitation channel
(due to suppression of dominant E1(T=1) motion)

irrotational

vortical

irrotational

$$E = 50 \div 60 A^{-1/3} \text{ MeV}$$

$$E = 50 \div 70 A^{-1/3} \text{ MeV}$$

$$E = 132 A^{-1/3} \text{ MeV}$$

Reviews:

N. Paar et al, Rep. Prog. Phys. 70 691 (2007);

D. Savran et al, Prog. Part. Nucl. Phys. 70, 210 (2013)

VON, J. Kvasil, A. Repko, W. Kleinig, and P.-G. Reinhard, Phys. Atom. Nucl. 79, 842 (2016).

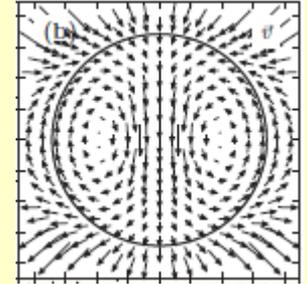
- Different kinds of dipole oscillations with fixed c.m.

Three conceptions of nuclear vorticity: HD, RW, toroidal

1. Hydrodynamical vorticity:

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \quad \delta \vec{v}(\vec{r}) = \frac{\delta \vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$\vec{\nabla} \times \delta \vec{v} = \frac{\rho_0 (\vec{\nabla} \times \delta \vec{j}_{nuc}) - \vec{\nabla} \rho_0 \times \delta \vec{j}_{nuc}}{\rho_0^2}$$



2. RW vorticity

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0 \quad \text{- continuity equation}$$

$$\delta \vec{j}_{1\mu}^v(\vec{r}) = \left\langle \nu \left| \hat{j}_{nuc}(\vec{r}) \right| 0 \right\rangle = -\frac{i}{\sqrt{3}} \left[\underbrace{j_{10}^v(r)}_{j_-} \vec{Y}_{10\mu}^* + \underbrace{j_{12}^v(r)}_{j_+} \vec{Y}_{12\mu}^* \right] \quad \text{- current transition density}$$

$$j_+^v(r)$$

- independent part of charge-current distribution,
- decoupled from CE in the integral sense
- **may be a measure of vorticity**

$$\nabla \cdot (j_+^v(r) \vec{Y}_{12\mu}^*) \neq 0$$

3. Toroidal strength as a measure of vorticity

P.-G. Reinhard et al, PRC 89, 024321
(2014)

Toroidal vortical mode appears in:

★ nuclear **current** density

Following theorems of Helmholtz and Chandrasekhar/Moffat, the current distribution can be decomposed as

V.M. Dubovik and A.A. Cheshkov, Sov. J. Part. Nucl. v.5, 318 (1975).

$$\vec{j}(\vec{r}) = \vec{\nabla} \phi(\vec{r}) + \vec{\nabla} \times [\vec{r} \psi(\vec{r})] + \vec{\nabla} \times \vec{\nabla} \times [\vec{r} \chi(\vec{r})]$$

electric moments

magnetic moments

electric **toroidal** moments

transversal

★ Multipole electric operator (**external** field) :

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}} \sqrt{\frac{\lambda}{\lambda+1}} \int d\vec{r} \hat{j}_{nuc}(\vec{r}) \cdot [\vec{\nabla} \times (\vec{r} \times \vec{\nabla}) (j_\lambda(kr) Y_{\lambda\mu})]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!} \left[1 - \frac{(kr)^2}{2(2\lambda+3)} + \dots \right]$$

So, the toroidal operator is the **second order** term in long-wave expansion of the electric operator

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r} \rho(\vec{r}) r^\lambda Y_{\lambda\mu} \leftarrow \begin{array}{l} \text{standard electric operator} \\ \text{In long wave approximation} \end{array}$$

Toroidal E1 operator:

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \left[r^3 + \frac{5}{3} r \langle r^2 \rangle_0 \right] \vec{Y}_{11\mu}(\hat{r}) \cdot \underbrace{[\vec{\nabla} \times \hat{j}_{nuc}(\vec{r})]}_{\text{mainly vortical flow}}$$

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c} \int d\vec{r} \left[r^3 - \frac{5}{3} r \langle r^2 \rangle_0 \right] Y_{1\mu} \underbrace{[\vec{\nabla} \cdot \hat{j}_{nuc}(\vec{r})]}_{\text{irrotational flow}} \quad \dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$



$$\hat{M}'_{com}(E1\mu) = \int d\vec{r} \hat{\rho}(\vec{r}) \left[r^3 - \frac{5}{3} r \langle r^2 \rangle_0 \right] Y_{1\mu}$$

Toroidal and compression operators are coupled:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2\sqrt{3}c} \int d\vec{r} \hat{j}_{nuc}(\vec{r}) \cdot \vec{\nabla} \times (\vec{r} \times \vec{\nabla}) \left[r^3 - \frac{5}{3} r \langle r^2 \rangle_0 \right] Y_{1\mu}(\hat{r})$$

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

Toroidal E1 resonance

- modern theoretical and experimental status
- TDR vs PDR

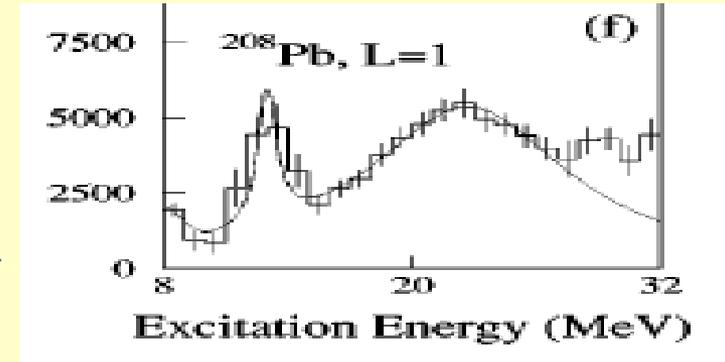
TDR and CDR constitute low- and high-energy ISGDR branches (?)

Experiment: (α, α')

- ^{208}Pb
- D.Y. Youngblood et al, 1977
 - H.P. Morsch et al, 1980
 - G.S. Adams et al, 1986
 - B.A. Devis et al, 1997
 - H.L. Clark et al, 2001
 - D.Y. Youngblood et al, 2004
 - M.Uchida et al, PRC 69, 051301(R) (2004)

Familiar treatment \longrightarrow

LE (toroidal) HE (compression)



TDR CDR

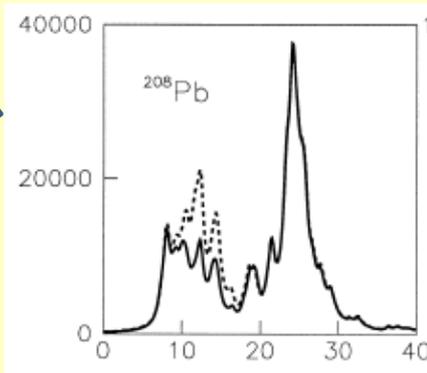
A.Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

There are also exp ISGDR data in

^{56}Fe , $^{58,60}\text{Ni}$, ^{90}Zr , ^{116}Sn , ^{144}Sm , ...

Theory:

- G. Colo et al, PLB 485, 362 (2000)
- D. Vretenar et al, PRC, 65, 021301(R) (2002)
- N. Paar et al, Rep. Prog. Phys. 70 691 (2007);



Perhaps Uchida observed at 10-17 MeV not TDR but CDR fraction coupled to TDR. Main TDR peak should lie lower at ~ 7-9 MeV.

The direct observation of TDR in (α, α') can be disputed in general since (α, α') is mainly determined by transition density while toroid mainly depends on the vortical transition current.

NEED IN NEW EXPERIMENTS!

Skyrme QRPA calculations

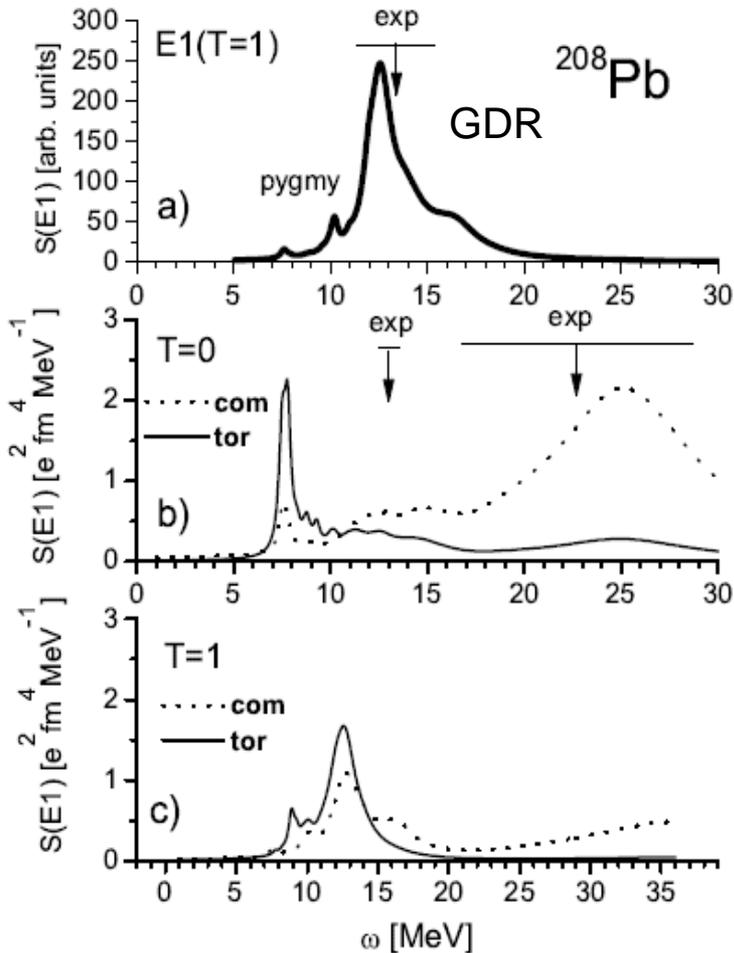
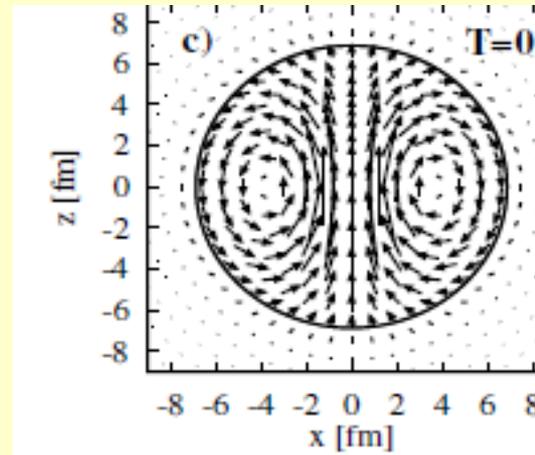
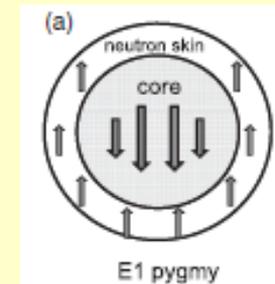
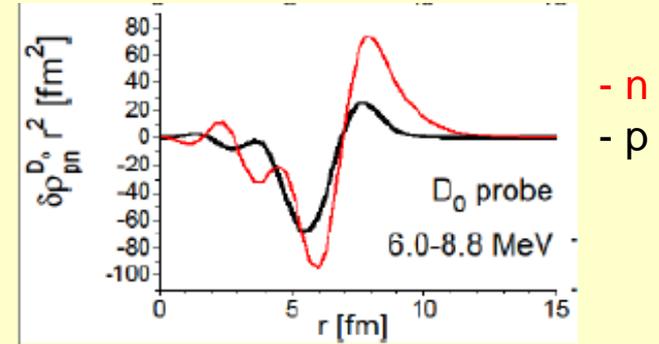
Strength functions

Sly6

A. Repko, P.G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013)

PDR region hosts TDR and CR!

Typical PDR transition density:



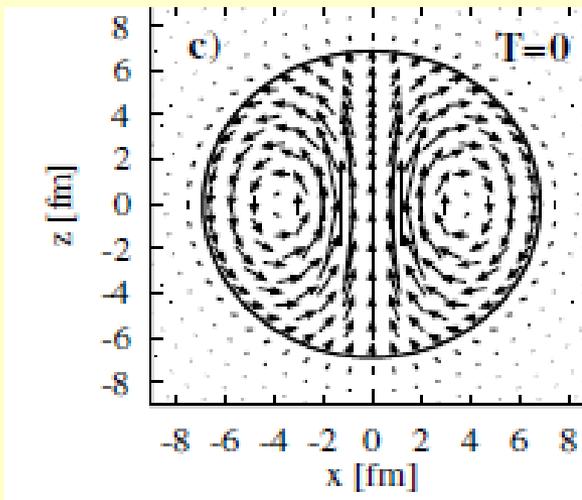
Nucleon current in the PDR region is mainly toroidal!



Toroidal flow in PDR energy region is obtained in various nuclei and within different models

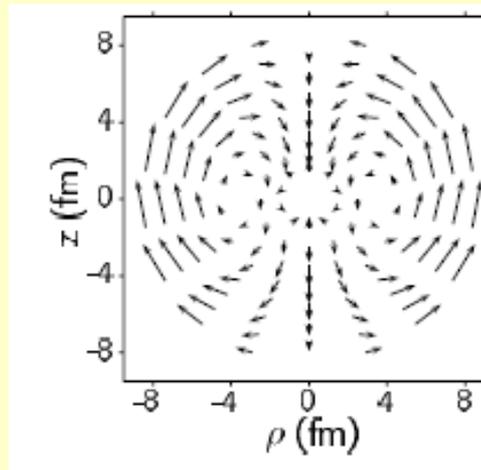
Skyrme RPA: 208Pb

Repko, P.-G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013).



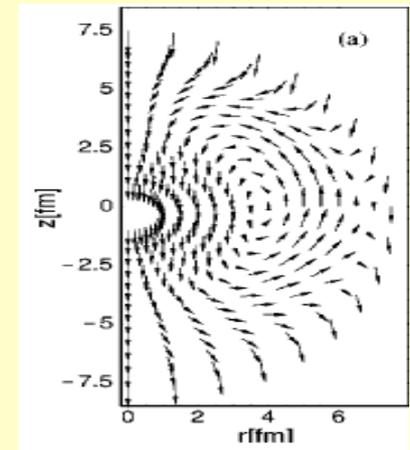
QPM: 208Pb

N.Ryezayeva et al,
PRL 89, 272502 (2002).



Relativistic RPA: 116Sn

D. Vretenar et al,
PRC 65, 021301R (2002).



Similar results in Ca, Ni,
Zr, Sn, Sm, Yb, U, ..

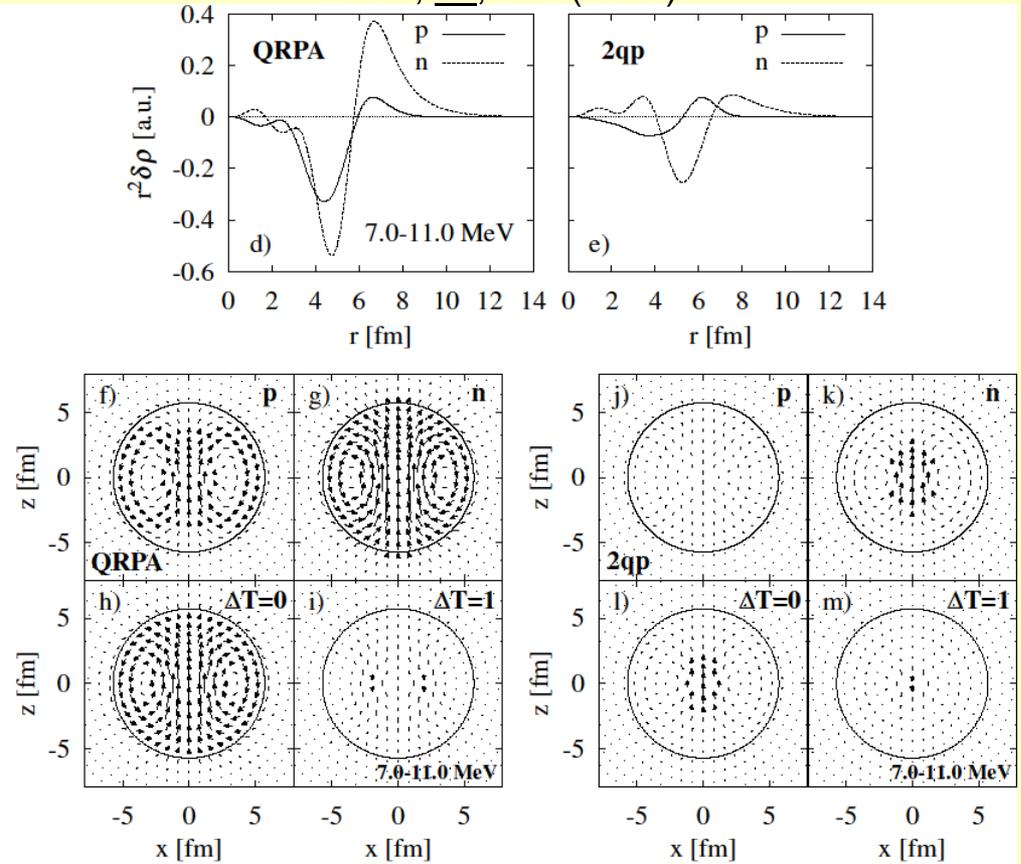
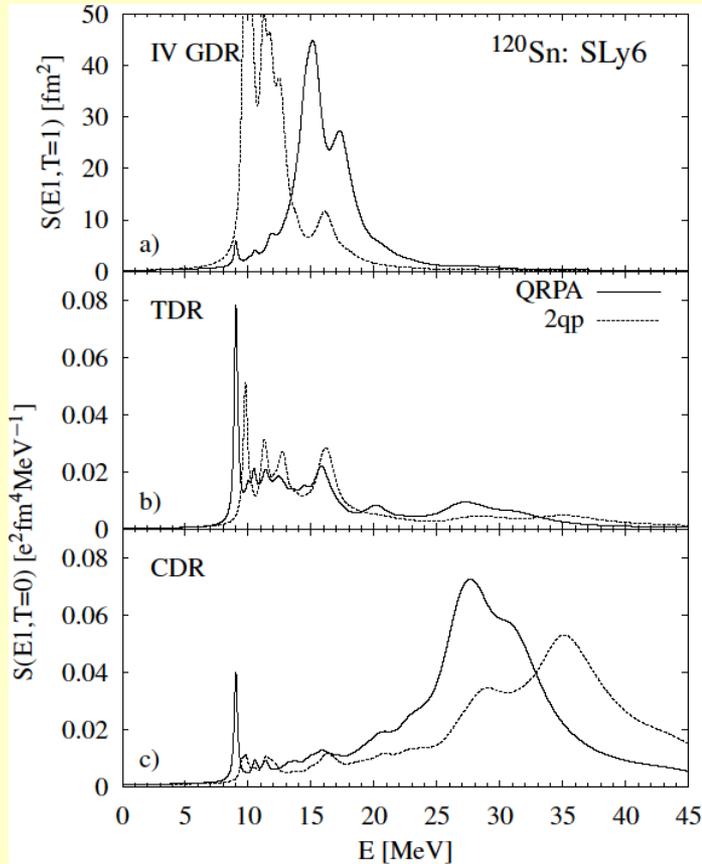
P. Papakonstantinou et al,
EPJA 47, 14 (2011).

Nuclear vorticity was also earlier discussed in:

F.E. Serr, T.S. Dumitrescu, T. Suzuki, C.H. Dasso, NPA 404, 359 (1983),
D.G. Ravenhall and J. Wambach, NPA 475, 468 (1987).

120Sn SLy6

A. Repko, VON, J. Kvasil and P.-G. Reinhard, EPJA, 55, 242 (2019)



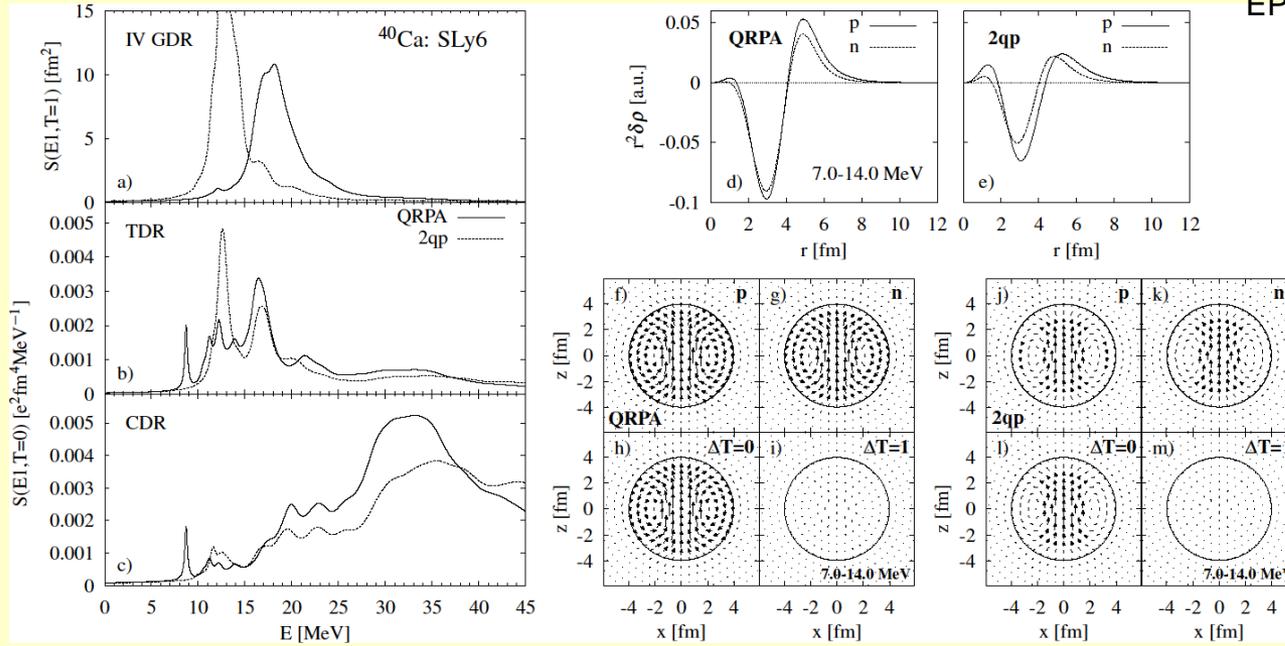
Strengths: TDR shares the same energy region with PDR and 2qp dipole strength
 RPA transition densities: typical for PDR
 RPA currents: clear IS toroidal flow with dominant neutron contribution

Is there one-to-one correspondence between transition densities and currents?

This can be checked through the continuity equation (CE): $-imE_\nu \delta\rho_\nu = \hbar^2 \vec{\nabla} \cdot \delta \vec{j}_\nu$
 Only irrotational part of the current with non-zero divergence contributes to CE.

40Ca, SLy6

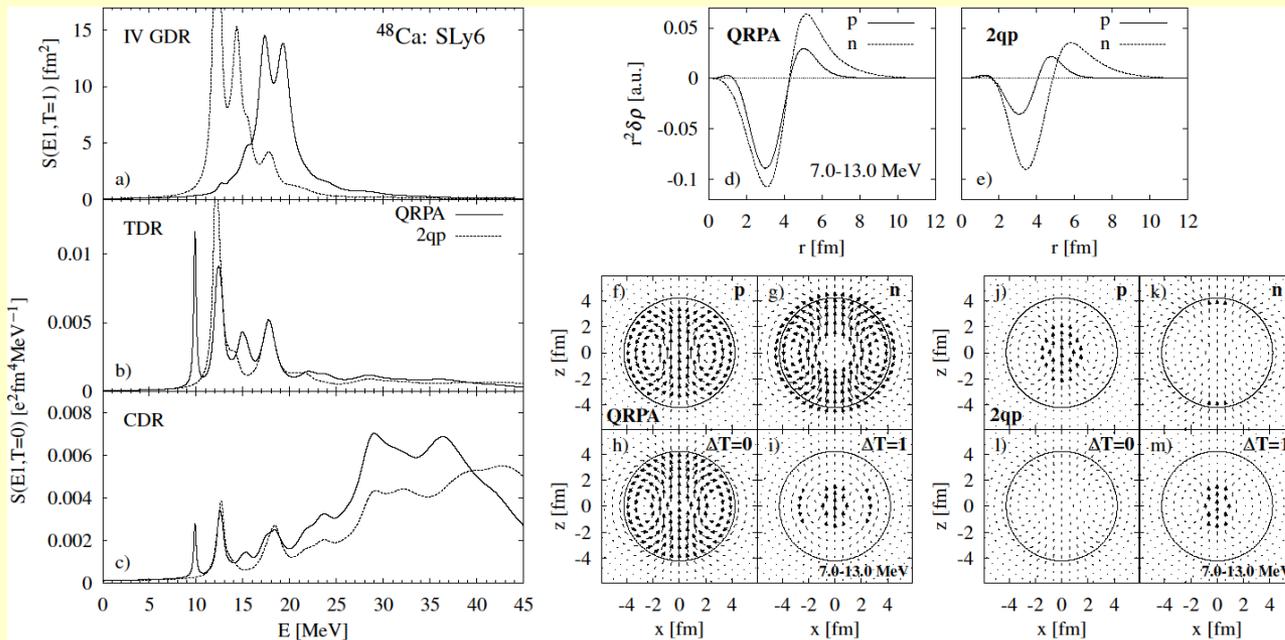
- no PDR but clear TDR
- residual interaction enforces toroidal flow
- protons and neutrons equally contribute
- squeezed toroidal flow

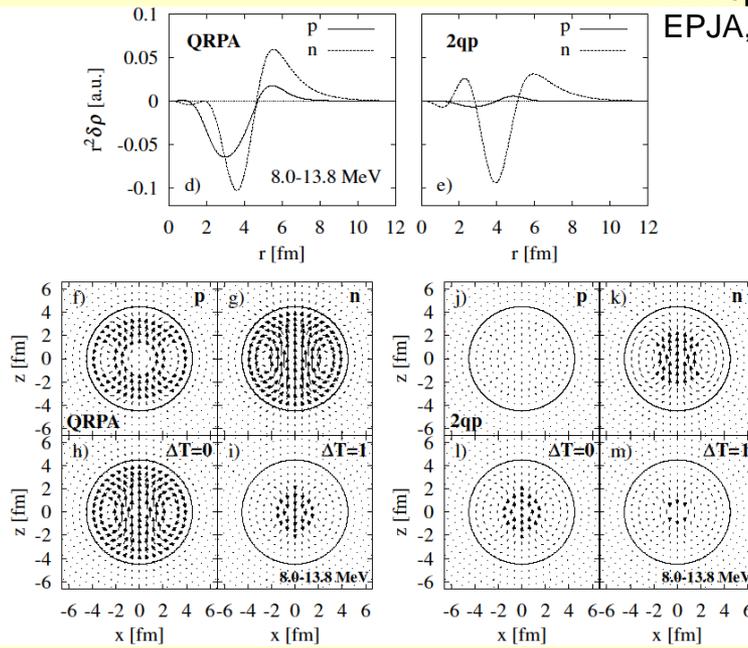
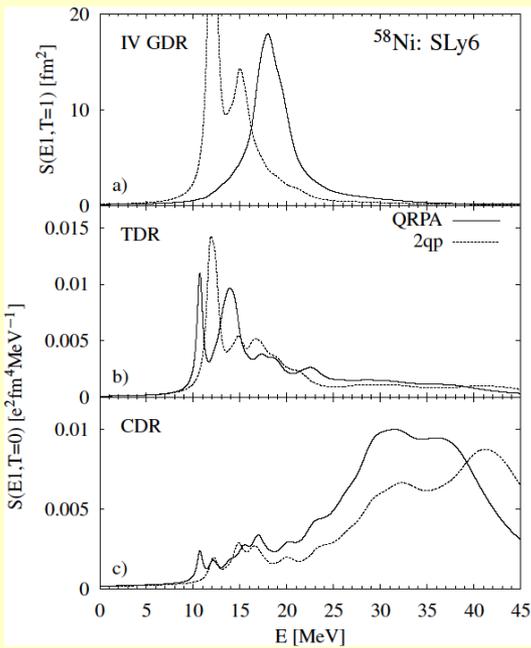


48Ca, SLy6

Basically the same results as for 120Sn, SLy6.

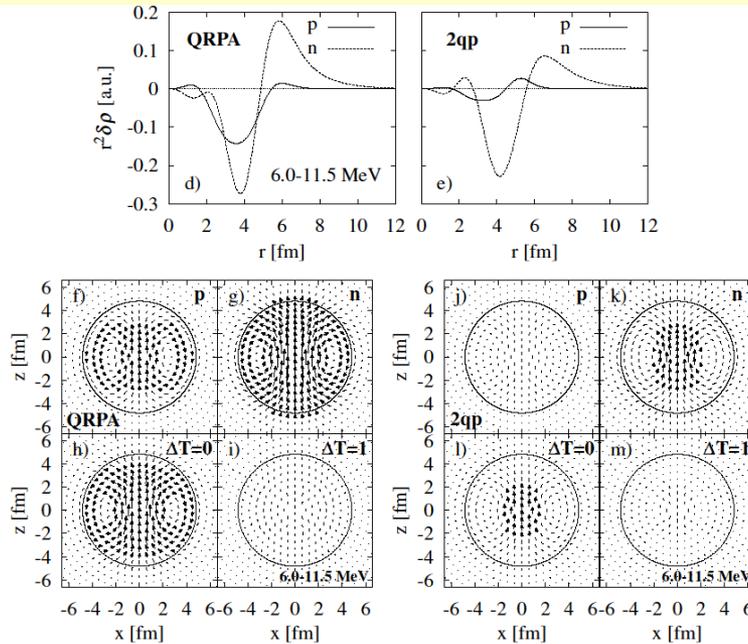
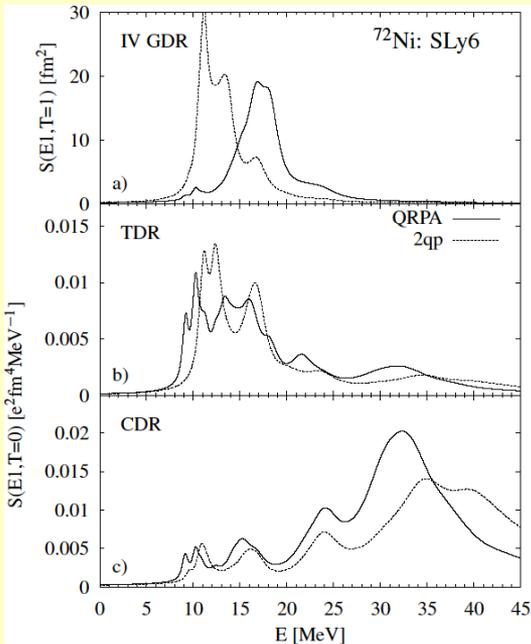
**Toroidal mode persists
In nuclei independently
on the neutron excess**





58Ni, SLy6

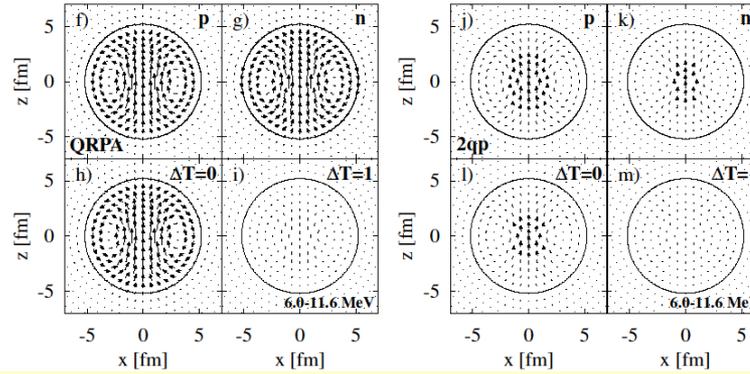
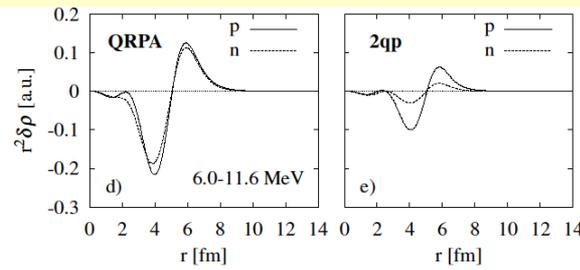
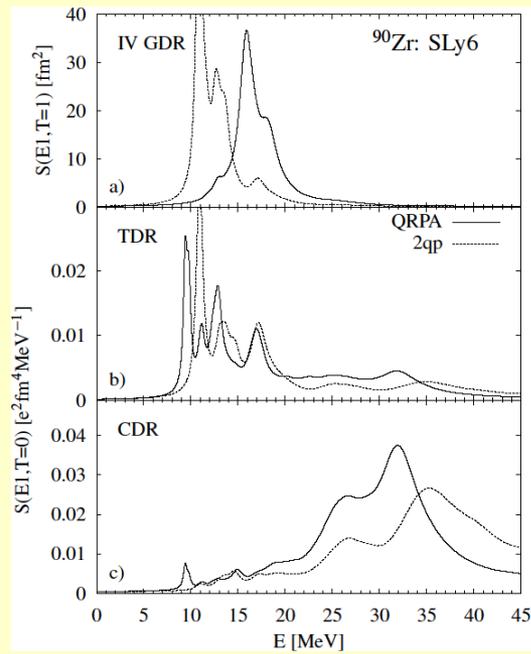
- small neutron excess, almost no PDR but strong TDR



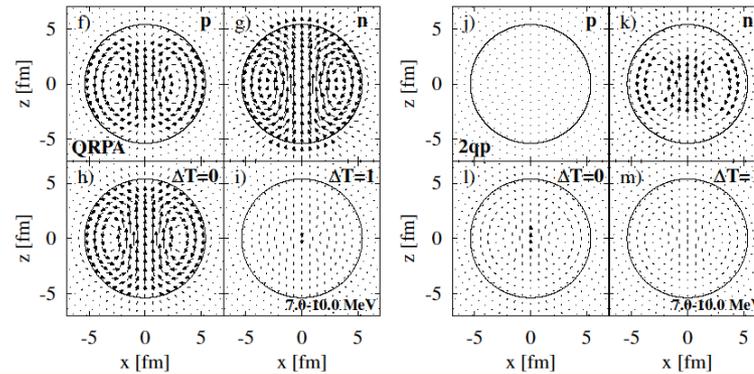
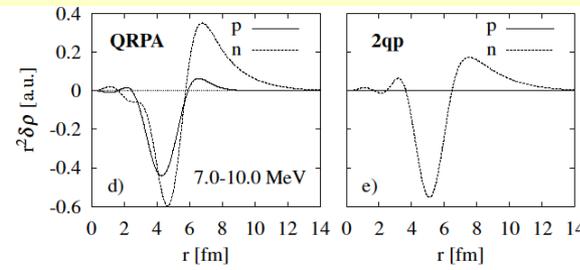
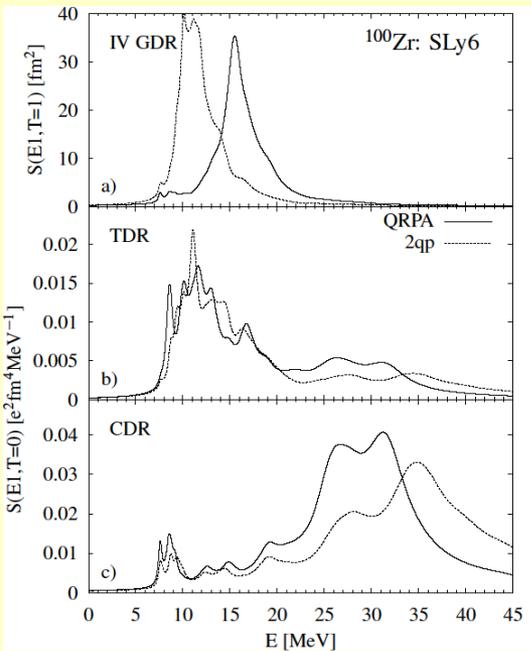
72Ni, SLy6

- large neutron excess
- dominant neutron, contribution to toroidal current,
- RPA enforces the flow
- basically the same results as for 120Sn, SLy6.

90Zr, SLy6



100Zr, SLy6



Basically the same results
 as for ^{120}Sn , SLy6.

Conclusions for TDR/PDR relation

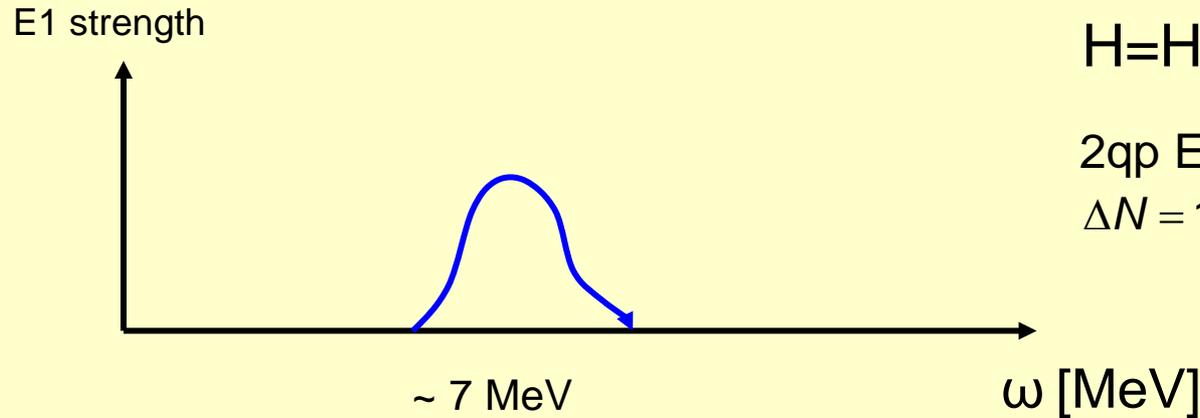
- The vortical E1 toroidal mode exists in all nuclei, independently on neutron excess.
- Dipole states in the PDR region have both:
 - i) dominant vortical fraction which gives TM current
 - ii) minor irrotational fraction which gives PDR-like transition density

$$\vec{j}(\vec{r}) = \underbrace{\vec{\nabla} \phi(\vec{r})}_{\text{PDR}} + \underbrace{\vec{\nabla} \times \vec{\nabla} \times [\vec{r} \chi(\vec{r})]}_{\text{TDR}}$$

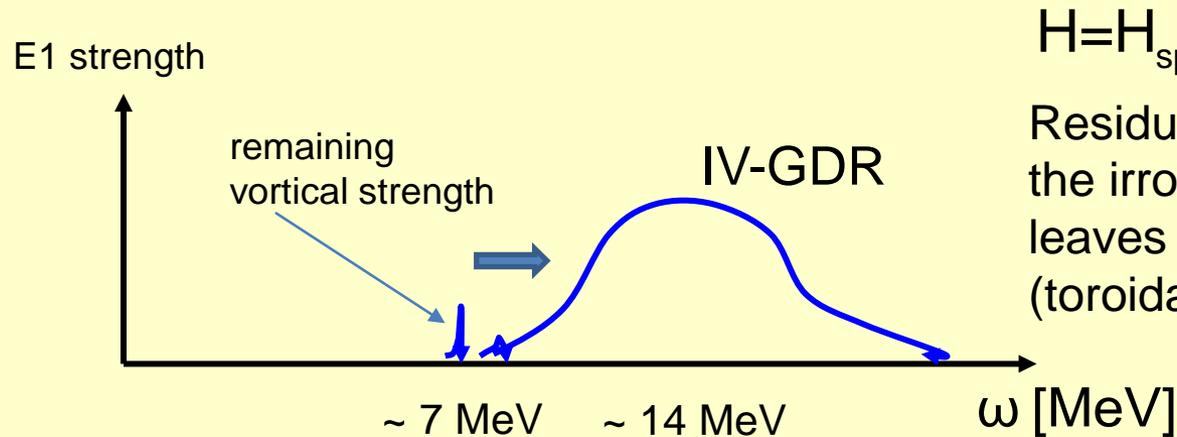
- PDR picture is a rough imitation of the actual (basically vortical) nuclear current.
- In many surface reaction, irrotational PDR but not TDR is excited.
- However, a minor PDR fraction can be used as a **doorway to generate TDR**.

Origin of E1 vortial toroidal strength

E1 toroidal strength must exist in all nuclei at the energy $\sim E(\Delta N = 1)$.



2qp E1 strength from
 $\Delta N = 1$ dipole transitions



Residual interaction upshifts
the irrotational E1 strength but
leaves at 7 MeV the vortical
(toroidal) strength.

Individual toroidal states in light nuclei

PHYSICAL REVIEW LETTERS **120**, 182501 (2018)

Individual Low-Energy Toroidal Dipole State in ^{24}Mg

V. O. Nesterenko,^{1*} A. Repko,² J. Kvasil,³ and P.-G. Reinhard⁴

¹*Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia*

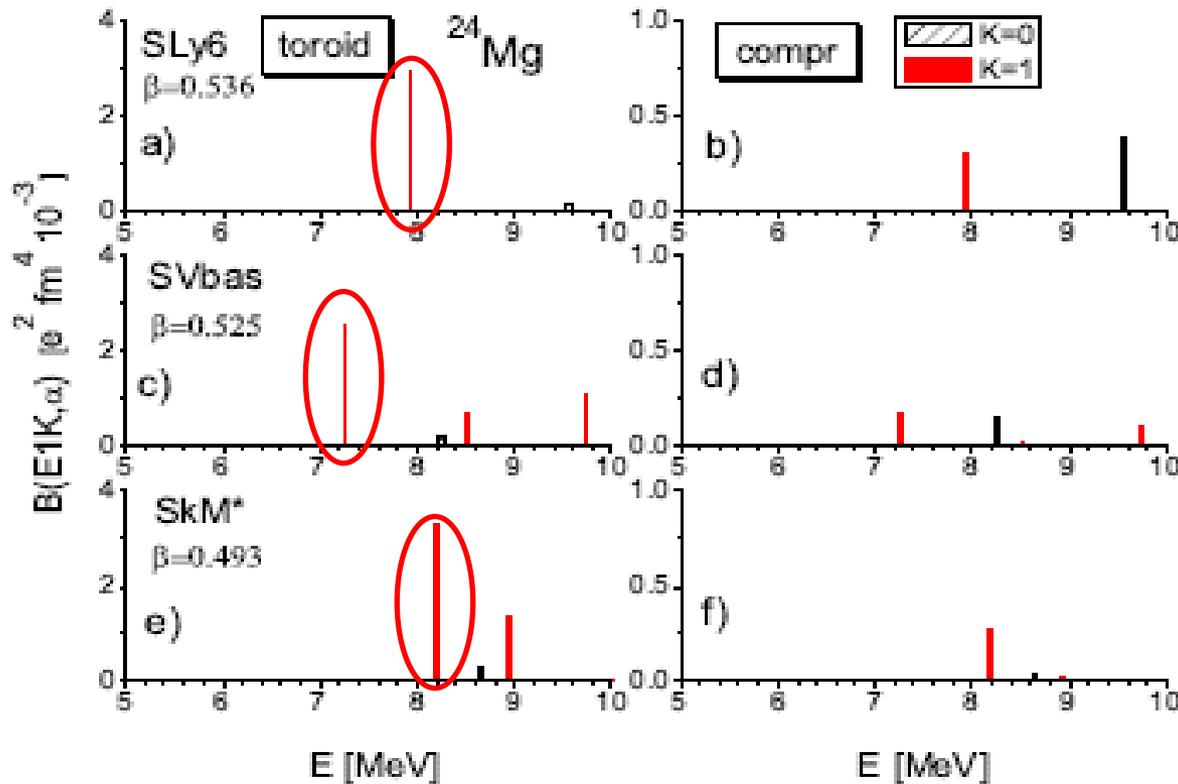
²*Department of Nuclear Physics, Institute of Physics SAS, 84511 Bratislava, Slovakia*

³*Institute of Particle and Nuclear Physics, Charles University, CZ-18000 Prague, Czech Republic*

⁴*Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany*

^{24}Mg

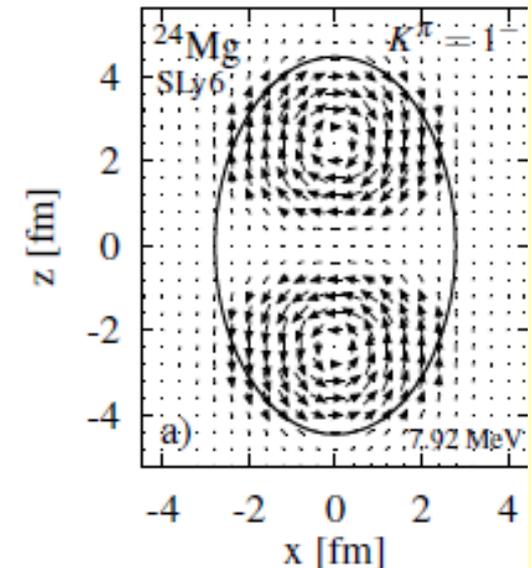
$$\beta_2^{\text{exp}} = 0.605$$



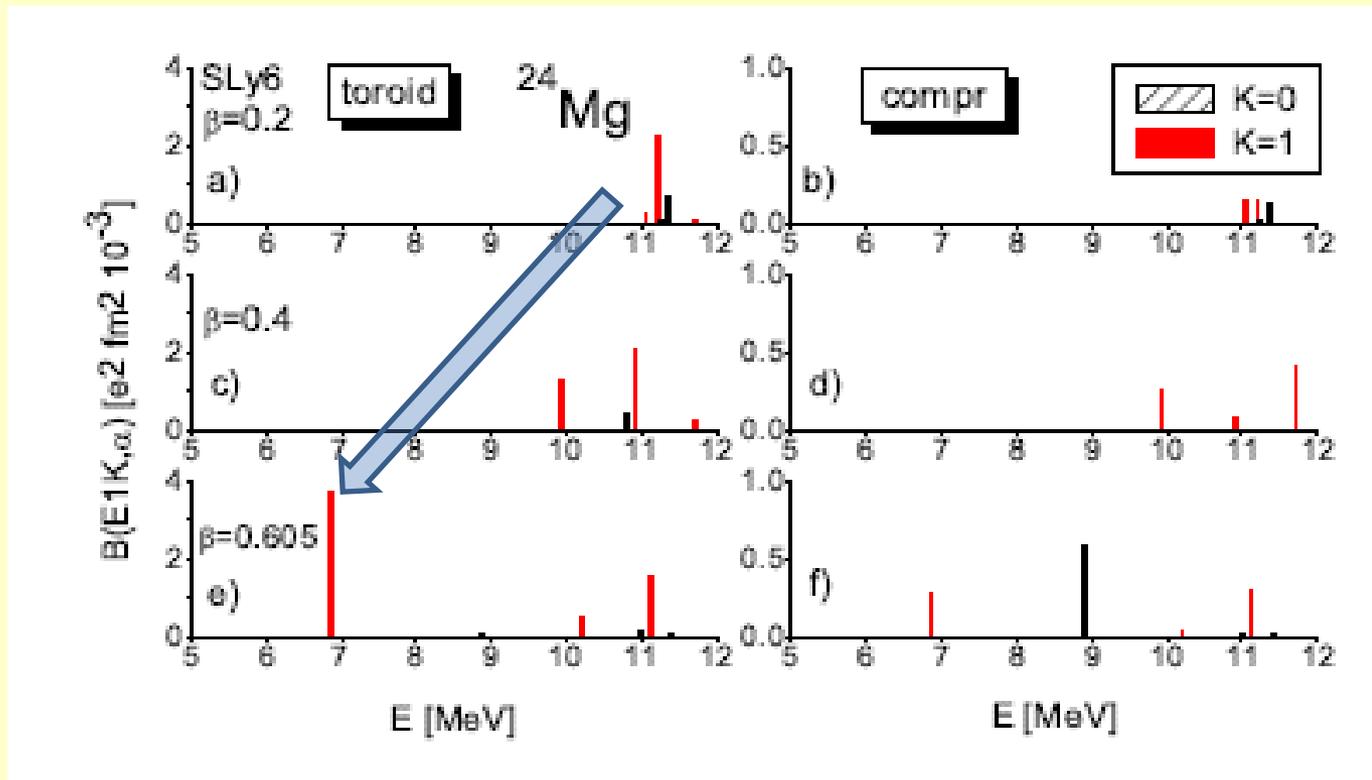
QRPA results for
SLy6,
SVbas,
SkM*

Persistence of the main result:
the **lowest** toroidal $K=1$ peak

The remarkable example of
individual toroidal state!



Dependence on deformation



TS becomes lowest due to of the large axial prolate deformation.

K=1 peak is:

- the lowest dipole state
- well separated from other states

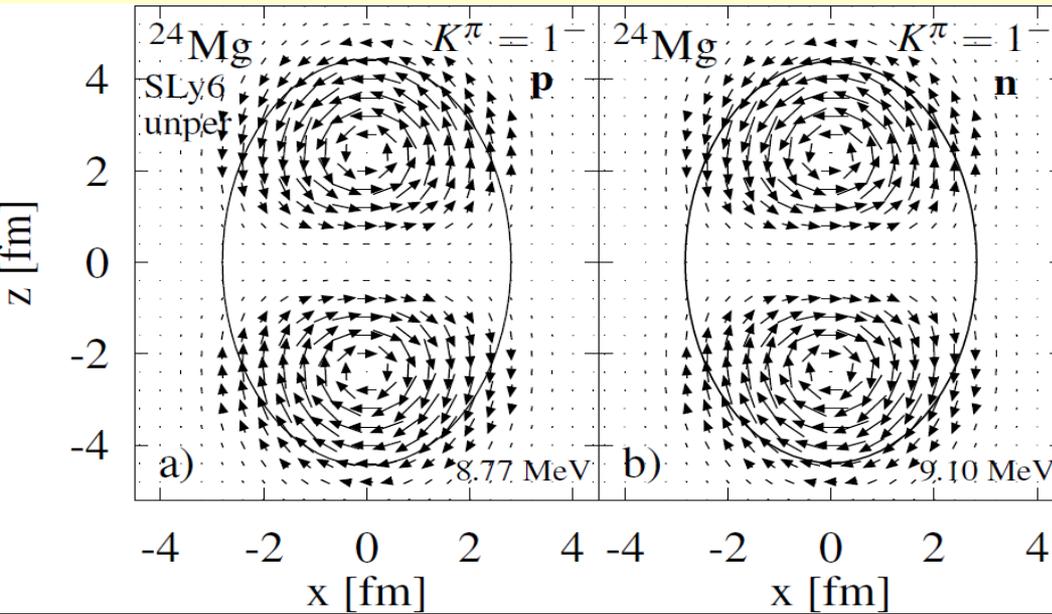
To get individual lowest TS, two rigorous requirements should be held:

- huge prolate deformations
- sparse low-energy spectrum

This is just realized in light deformed nuclei

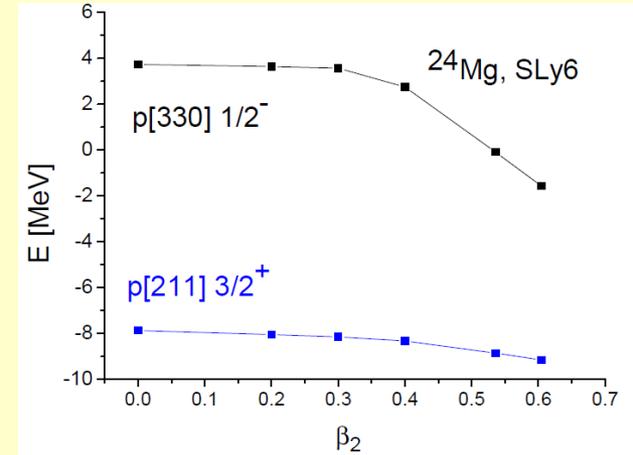
Toroidal flow: collective or 2qp origin?

flows of main 2qp configurations



$$pp[211] \uparrow - [330] \uparrow \quad (54\%)$$

$$nn[211] \uparrow - [330] \uparrow \quad (39\%)$$



Toroid is mainly 1ph (mean field) effect!

D.G. Ravenhall and J. Wambach,
NPA, 475, 468 (1987).

The deformation-induced energy downshift is not universal.

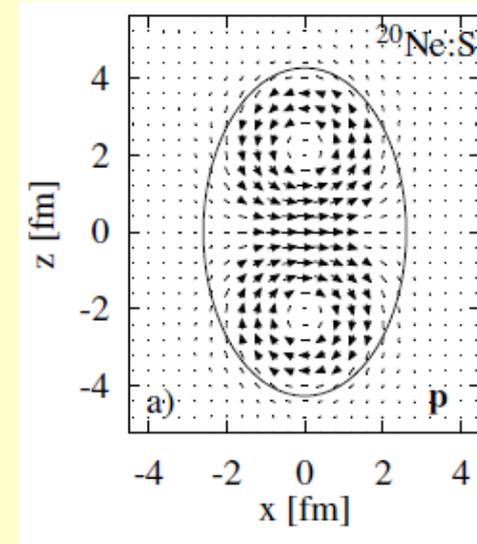
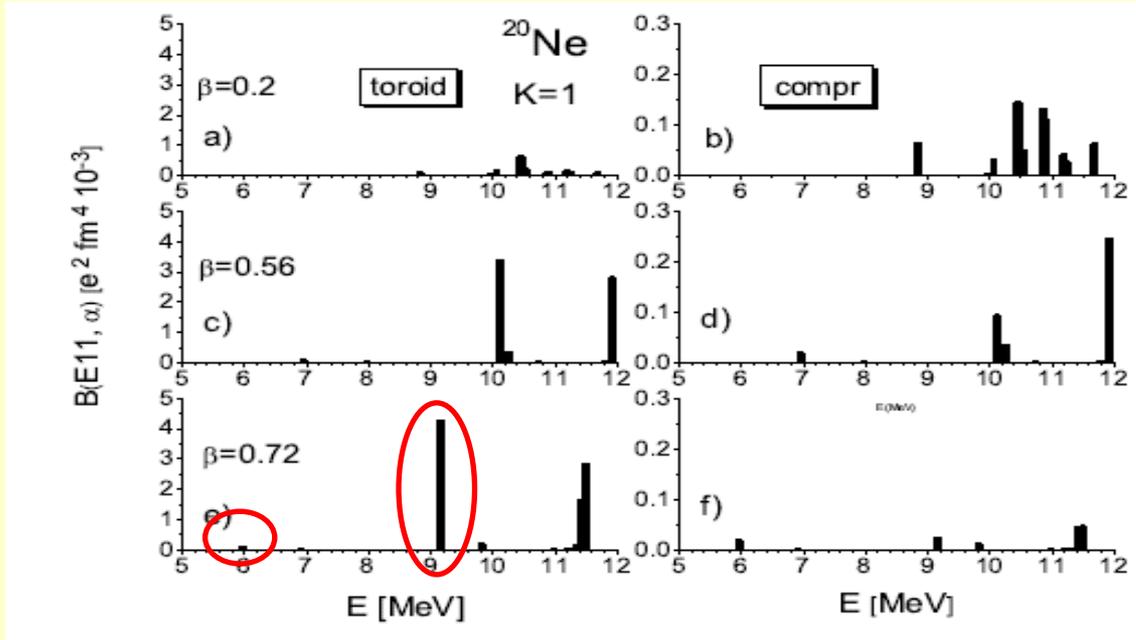
Perhaps ^{24}Mg is one of very few nuclei where the toroidal mode is the lowest dipole $K=1$ state.

Explains the deformation effect in TS

20Ne

$$\beta_2^{\text{exp}} = 0.72$$

V.O. Nesterenko, J. Kvasil, A. Repko, and P.-G. Reinhard, Eur. Phys. J. Web of Conf. 194, 03005 (2018) (2018)



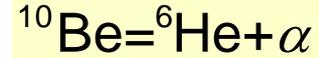
TM: not lowest

P. Adsley, VON, M. Kimura, L.M. Donaldson, R. Neveling, et al, PRC 103, 044315 (2021)

Interplay of **cluster and vortical modes** in **IS1 and IS0 states** in light nuclei with a **different deformation** (prolate 24Mg, soft 26Mg, oblate 28Si).

It was shown that **low-energy vorticity** is well localized in 24Mg, fragmented in 26Mg, and absent in 28Si.

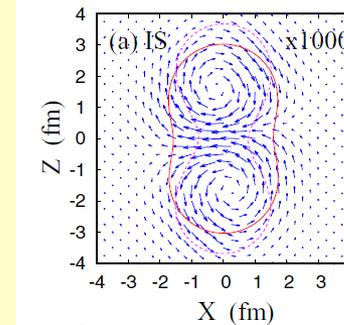
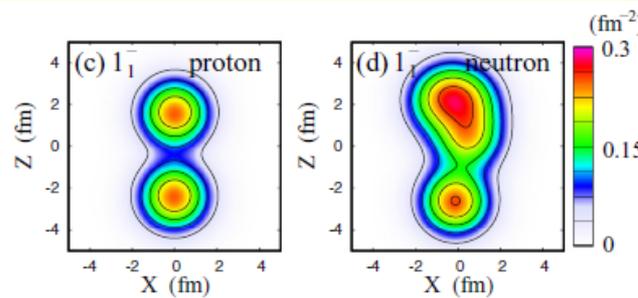
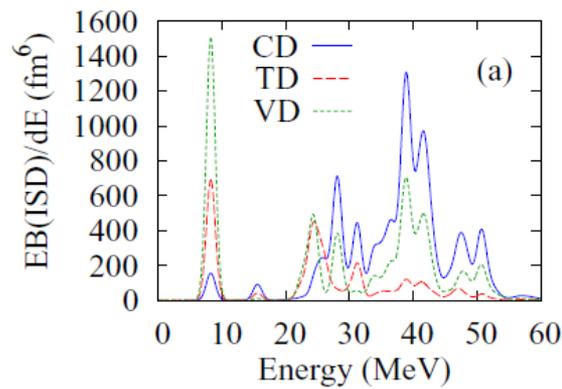
Toroidal, compressive, and $E1$ properties of low-energy dipole modes in ^{10}Be



Yoshiko Kanada-En'yo and Yuki Shikata

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

**Antisymmetrized molecular dynamics + generator coordinate method (AMD+GCM)
Cluster degrees + mean field**



the lowest dipole state
 $I^\pi K = 1^- 1_1$ is toroidal

Y. Kanada-En'yo, Y. Shikata, and H. Morita, Phys. Rev. C 97, 014303 (2018)

Y. Kanada-En'yo and H. Horiuchi, Front. Phys. 13, 132108 (2018)

Y. Kanada-En'yo, Y. Shikata, and H. Morita, PRC 97, 014303 (2018)

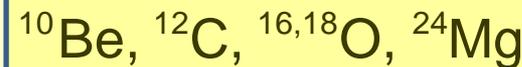
Y. Shikata, Y. Kanada-En'yo, and H. Morita, Prog. Theor. Exp. Phys. **2019**, 063D01 (2019).

Y. Kanada-En'yo and Y. Shikata, Phys. Rev. C 100, 014301 (2019).

Y. Shikata and Y. Kanada-En'yo, PRC, 103, 034312 (2021).

Y. Chiba, Y. Kanada-En'yo, and Y. Shikata, arXiv:1911.08734.

In AMD+GCM, toroidal states were found in



Interplay of cluster and vortical modes:

So,

- existence of individual toroidal states (ITS) is confirmed by different models (Skyrme QRPA and cluster AMD+GCM)
- ITS exist in all light nuclei below α -particle threshold
- in deformed ^{24}Mg , ITS is the lowest dipole state with $K=1$
- ITS can be easier observed and identified than TDR

Search of TDR in (e, e')

TM can be excited in various reactions through its irrotational fraction as the doorway:

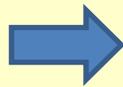
1) Isoscalar reactions (α, α') , (d, d') , $(^{17}\text{O}, ^{17}\text{O}')$

2) (e, e')

Most simple reaction to start the theoretical analysis

3) Reactions of inelastic scattering with γ -decay

$(e, e' \gamma)$
 $(\alpha, \alpha' \gamma)$



scattering angle of the photon can depend on the nuclear flow.

In analogy with relativistic heavy ion collisions (HIC), a vortical flow can lead to polarization of the outgoing γ .

General problem:

- modern theory and experiment are not yet able to propose ways for identification of **intrinsic vortical** modes. This fundamental problem is still unresolved

Reviews of experimental methods:

D. Savran, T. Aumann, and A. Zilges, Prog. Part. Nucl. Phys., 70, 210 (2013)

A. Bracco, E.G. Lanza, and A. Tamii, Prog. Part. Nucl. Phys., 106, 360 (2019)

(e,e'): PWBA cross section

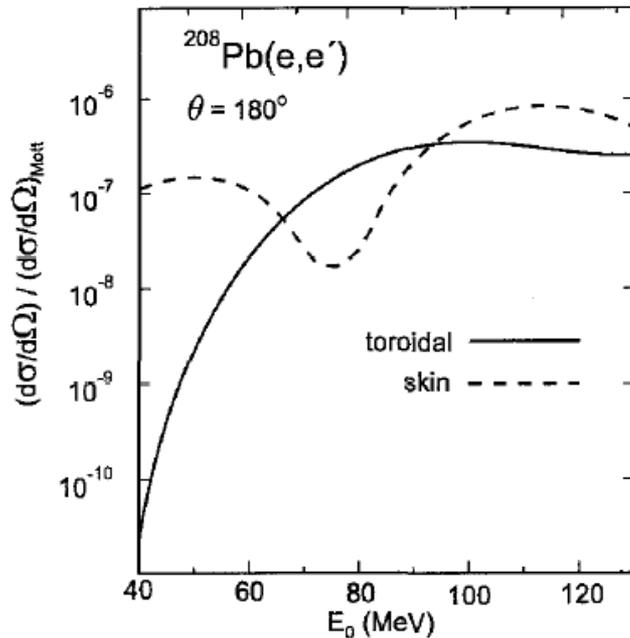
H. Theissen, Springer Tacts in Mord. Phys., 65, 1 (1972)

J. Heisenberg and H.P. Blok, Ann. Rev. Nucl. Part. Sci, 33, 569 (1983),

$$\sigma_{PWBA}(\theta, q) = \sigma_{Mott}(\theta, E_i) f_{rec} \left\{ |F_E^C(q)|^2 + \left(\frac{1}{2} + \text{tg}^2\left(\frac{\theta}{2}\right) \right) [|F_E^T(q)|^2 + |F_M^T(q)|^2] \right\}$$

For $I^\pi = 1^-$ states, $F_M^T(q) = 0$ but \hat{j}_{mag}^q contributes to $F_E^T(q)$

A. Richter / Nuclear Physics A731 (2004) 59–75



A. Richter, NPA, 731, 59 (2004).

Toroidal mode contributes to E1 transversal form-factor and can in principle be discriminated in (e,e') to back angles.

Here we meet the problem: impact of the magnetization current

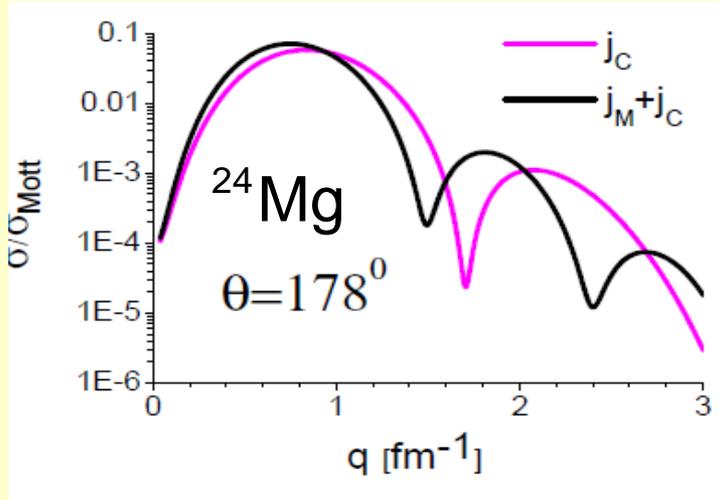
$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} [r^3 + \frac{5}{3}r < r^2 >_0] \vec{Y}_{11\mu}(\hat{r}) \cdot [\vec{\nabla} \times \hat{j}_{nuc}(\vec{r})]$$

Nuclear current

$$\hat{j}_{nuc}(\vec{r}) = \frac{e\hbar}{m} \sum_{q=n,p} (\hat{j}_{con}^q(\vec{r}) + \hat{j}_{mag}^q(\vec{r}))$$

$$\hat{j}_{con}^q(\vec{r}) = -ie_{eff}^q \sum (\delta(\vec{r} - \vec{r}_k) \vec{\nabla}_k - \vec{\nabla}_k \delta(\vec{r} - \vec{r}_k)) \rightarrow \text{toroidal flow}$$

$$\hat{j}_{mag}^q(\vec{r}) = \frac{g_s^q}{2} \gamma \sum_{k \in q} \vec{\nabla}_k \times \hat{s}_{qk} \delta(\vec{r} - \vec{r}_k), \quad \gamma = 0.7$$



PWBA for 24Mg

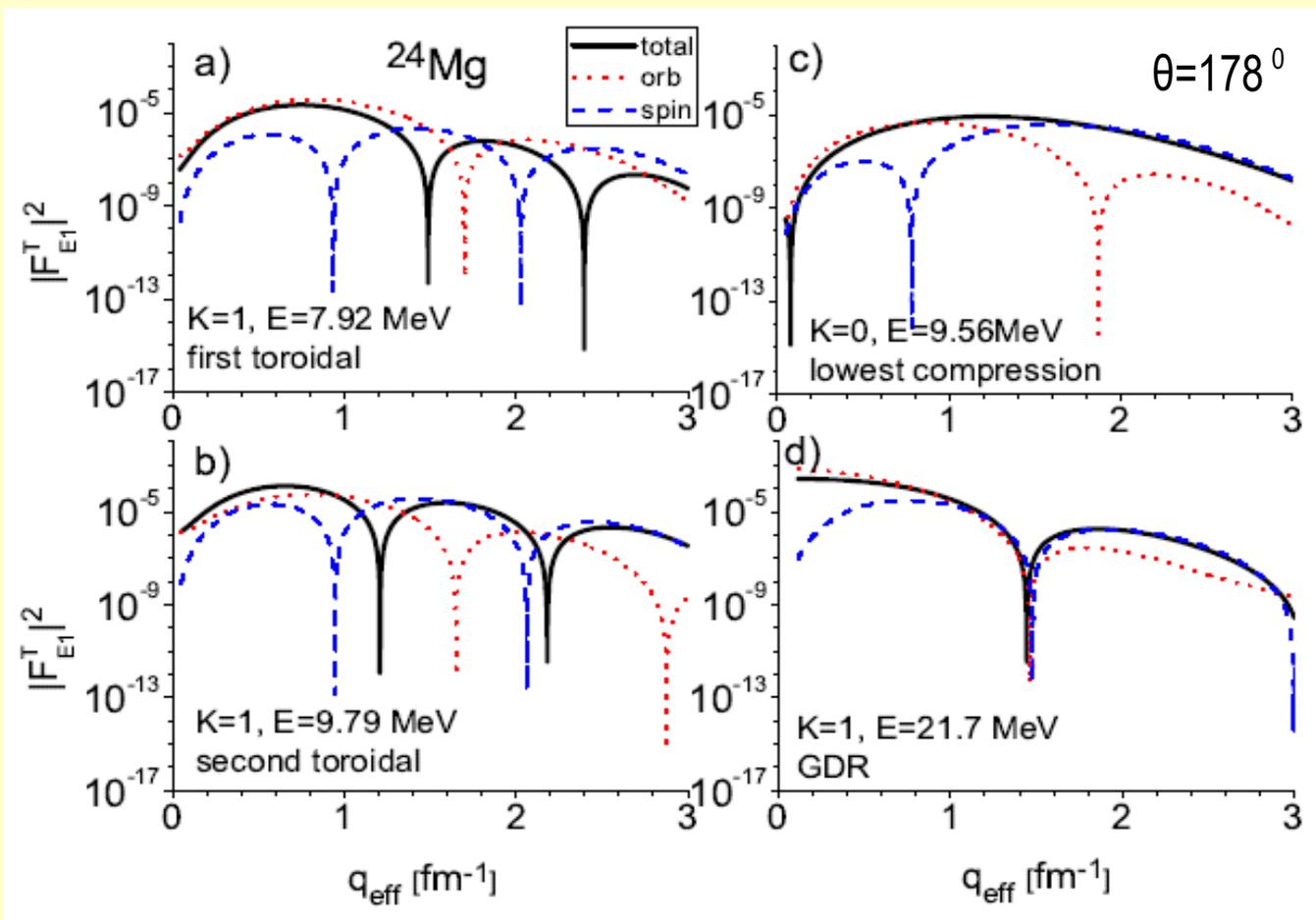
Impact of \vec{j}_M current is significant at $q > 1 \text{ fm}^{-1}$

Makes toroidal effect unresolved?

It will be shown that \vec{j}_M can even help in search of E1 TM!

Transversal E1 form factor for different dipole states: orbital and spin contributions

V.O. Nesterenko et al,
PRC 100, 064302 (2019).



In toroidal states,

behavior of $|F_{E1}^T|^2$ is determined by the **strong interference** of \vec{j}_C and \vec{j}_M contributions. None of these contributions alone can describe $|F_{E1}^T|^2$!

The spin/orbit interference in E1 and M2 form factors can be used for confirmation of the toroidal nature of the state.

Two-step scheme to search toroidal states (TS) in (e, e') can be proposed:

- 1) The appropriate candidates for TS have to be chosen from e.g. QRPA calculations for E1(K=1) states.

These candidates should have:

- significant toroidal E1 strength
- clear toroidal distribution of the nuclear current
- enhanced $B(M2, K=1)$ value (for deformed nuclei)

For the chosen states, the form factors F_{E1}^T and F_{M2}^T to back scattering are calculated.

- 2) If the calculated form factors well describe exp data, then we may be confident that we correctly reproduce the vortical orbital and spin fractions and their interference.

Then we may be confident that the toroidal distribution of the nuclear current is also realistic.

Conclusions

★ TDR is a remarkable example of the **vortical intrinsic electric** nuclear flow. TDR is the **general feature** of atomic nuclei, Exploration of the vortical flow in nuclei is yet very poor. Study of TDR can be a first important step in solution of this problem.

★ TDR vs PDR

**TDR coexists with PDR.

** PDR picture (oscillations of the neutron excess against the core) is a rough imitation of the actual (basically toroidal) flow.

** PDR is formed by a **small irrotational fraction** of mainly vortical dipole states.

** PDR can be used as a doorway state in excitation of TDR.

★ Individual toroidal states (ITS) in light nuclei as a new way to explore vortical excitations. Interplay of cluster and vortical modes.

★ Search of TDR and ITS in (e, e') using the interference pattern from contributions of convection and magnetization parts of nuclear current.

Outlook: search of ITS in $(e, e' \vec{\gamma})$, sum rules, similarities with HIC, ...

Thank you for attention!